Formal Definition of a Limit
Demonstration Activity

1) Open the Formal Definition of a Limit Demonstration located at http://demonstrations.wolfram.com/FiniteLimitAtAFinitePoint/

2) First get familiar with the various options. Adjust the slider for ϵ. Uncheck the box for calculating the automatic δ values. You can move the vertical line for c and the orange lines for δ using your mouse.

3) Center the purple line for $c = 5$.

4) Set $\epsilon = .5$

5) Make sure to uncheck the box for automatically calculating δ. Move the orange lines so that they intersect the function as close to $L - \epsilon$ and $L + \epsilon$ as possible. $f(x)$ should now be bounded: $L - \epsilon \leq f(x) \leq L + \epsilon$. List the two δ values you end up with:

   $\delta_1 =$ ____________ $\delta_2 =$ ____________

6) What is the minimum δ above? min$(\delta_1, \delta_2) =$ ____________

7) Therefore, (fill in the blanks)

   If $|x -$ $\quad$ $| <$ $\quad$ then $|f(x) -$ $\quad$ $| <$

8) Click on the box to automatically calculate δ. min$(\delta_1, \delta_2) =$ ____________
   How close was your minimum δ to the minimum calculated δ?

9) Repeat this process for $c = 5$ and $\epsilon = .225$.

   $\delta_1 =$ ____________ $\delta_2 =$ ____________ min$(\delta_1, \delta_2) =$ ____________

   If $|x -$ $\quad$ $| <$ $\quad$ then $|f(x) -$ $\quad$ $| <$

10) Repeat this process for $c = 5$ and $\epsilon = .1$.

$\delta_1 =$ ____________ $\delta_2 =$ ____________ $\min(\delta_1, \delta_2) =$ ____________

If $|x - \quad | <$ then $|f(x) - \quad | <$

11) Repeat this process for $c = 3$ and $\epsilon = .5$.

$\delta_1 =$ ____________ $\delta_2 =$ ____________ $\min(\delta_1, \delta_2) =$ ____________

If $|x - \quad | <$ then $|f(x) - \quad | <$

12) Repeat this process for $c = 3$ and $\epsilon = .225$.

$\delta_1 =$ ____________ $\delta_2 =$ ____________ $\min(\delta_1, \delta_2) =$ ____________

If $|x - \quad | <$ then $|f(x) - \quad | <$

13) Repeat this process for $c = 3$ and $\epsilon = .1$.

$\delta_1 =$ ____________ $\delta_2 =$ ____________ $\min(\delta_1, \delta_2) =$ ____________

If $|x - \quad | <$ then $|f(x) - \quad | <$

14) Now we will replicate what this demonstration shows using your graphing calculator.

a) First, let $f(x) = x^2$. Find the $\lim_{x \to 2} f(x) =$ ____________ (this is L).

b) We want to know how close to $x = 2$ we need to get so that $f(x)$ is within $\boldsymbol{\epsilon = .1}$ of L.
On your calculator, let Y1 $= f(x)$, Y2 $= L - \epsilon$, Y3 $= L + \epsilon$.
Let xmin $= 1$, xmax $= 3$, ymin $= L - 2\epsilon$, ymax $= L + 2\epsilon$.
Graph.

Find where $f(x)$ intersects $L - \epsilon$. This $x -$ value with be $2 - \delta_1$. What is $\delta_1 =$ ________?

Find where $f(x)$ intersects $L - \epsilon$. This $x -$ value with be $2 + \delta_2$. What is $\delta_2 =$ ________?

What is the min$(\delta_1, \delta_2) =$ ______________?

Therefore, if $|x -$ $\quad$ $| <$ $\quad$ then $|f(x) -$ $\quad$ $| <$

15) Repeat this process for $f(x) = x^3 - x + 1$, $\epsilon = .01$ and $\lim_{x \to 1} f(x)$ using your calculator.

$\delta_1 =$ ____________ $\quad$ $\delta_2 =$ ____________ $\quad$ min$(\delta_1, \delta_2) =$ ____________

If $|x -$ $\quad$ $| <$ $\quad$ then $|f(x) -$ $\quad$ $| <$