Newton's Law of Cooling
Demonstration Activity

1) Open the Newton's Law of Cooling Demonstration located at
http://demonstrations.wolfram.com/TheCoffeeCoolingProblem/

2) First get familiar with the various options. Adjust the sliders to set the time to add cream to the coffee. Adjust the slider to begin time. You can animate it by pressing play.

3) Set the slider to add cream right away to cup 1. Play around with when to add the cream to cup 2. Animate what happens by pressing play for the time. Make a hypothesis for what to do in the following two situations: (Assume the initial temperature of the coffee is 185° and coffee is safe to drink at 143°.)

a) If you want to drink your coffee as soon as possible, when should you add the cream?

b) If you want to keep your coffee hot as long as possible - maybe you get called away just after pouring your coffee - when should you add the cream?

4) Add cream to cup 1 right away. Add cream to cup 2 at 200 seconds.

a) Which do you think will cool to 143° first, cup 1 or cup 2?

b) List the temperatures of the two cups at the indicated times in the table on the next page:

Time	Cup 1 °F	Cup 2 °F
50		
100		
150		
190		
195		
199		
200		
201		
205		
210		
220		
250		
300		
350		
400		
500		

c) Just before the cream is added to cup 2, how much warmer is cup 2?

d) How many seconds after the cream is added to cup 2 does the temperature of cup 2 go below cup 1?

e) How many seconds until cup 1 is at a drinkable temperature (143°F)?

f) How many seconds until cup 2 is at a drinkable temperature?

g) So, if you want the coffee to cool to drinking temperature as soon as possible, when should you add the cream?

h) So, if you want to keep the coffee as hot as possible, when should you add the cream?

5) Add cream to cup 1 right away. Add cream to cup 2 at 350 seconds.

a) Just seconds before adding cream to cup 2, list the temperatures of cup 1 and cup 2:

Time:_____ Cup 1:_____ Cup 2:_____

b) How many seconds after adding the cream to cup 2 does it get cooler than cup 1?

c) After how many seconds does cup 1 get drinkable?

d) After how many seconds does cup 2 get drinkable?

6) Now you will solve this Newton's Law of Cooling problem by hand. If you want to drink your coffee as soon as possible, then the optimal strategy is wait to add the cream. This is a consequence of Newton's Law of Cooling which says that the cooling rate is proportional to the difference between the temperature of the coffee and the ambient temperature. We will assume that the ambient temperature and the temperature of the cream are both 68°F. So the coffee will cool faster the greater the difference between the temperature of the coffee and the temperature of the cream.

a) The rate of change of the temperature of the coffee is proportional to the difference between the temperature of the coffee and the ambient temperature. This can be written as:

$$\frac{dT}{dt} = k(T - T_{\text{surrounding}})$$

The solution to this differential equation is $T(t) = (T_{\text{initial}} - T_{\text{surrounding}})e^{kt} + T_{\text{surrounding}}$.

What is T_{initial} in our problem?_____

What is $T_{\text{surrounding}}$ in our problem?_____

b) Find out how long it will take a black cup of coffee (no cream added) to cool to a drinkable temperature of 143°F. Assume that after 50 seconds, the temperature of the coffee was 181.2°. Verify your result using the demonstration.

c) Find out how long it will take a cup of coffee with cream added right away to cool to a drinkable temperature of 143°F. Since the cream cools the coffee almost immediately, assume that the initial temperature of the coffee with cream is 167° (this is T_{initial}). After 50 seconds, the temperature drops to 163.8°. Verify your result using the demonstration.