Absolute Maximums and Minimums

Abs. Maximum =

Abs. Minimum =

Continuous =

Abs. Maximum =

Abs. Minimum =

Continuous =

Abs. Maximum =

Abs. Minimum =

Continuous =

Extreme Value Theorem = If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

Notice that the max's and min's above can occur at endpoints or on the inside of the interval. These are called <u>absolute maxima</u> or <u>absolute minima</u> or <u>absolute extrema</u>.

Consider:

Absolute maxima =

Absolute minima =

Relative Maxima =

Relative minima =

Definition of Relative Extrema

1. If there is an open interval containing c on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum** of f, or f has a relative maximum at $(c, f(c))$.

2. If there is an open interval containing c on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum** of f, or f has a relative minimum at $(c, f(c))$.

Intuitively, a relative minimum would look like a valley and a relative maximum would look like a hill. If the hill (or valley) is smooth and rounded, then the graph has a horizontal tangent line at the extrema. If the hill (or valley) is sharp and peaked, the graph is not differentiable at the extrema.

The maximum and minimum of a function on an interval are called **absolute maximum** or **absolute minimum**. These may or may not exist on the interval. However, the following theorem guarantees that at least one of each will exist:

The Extreme Value Theorem:
If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

Absolute maxima and absolute minima can occur at the same points where relative maxima and minima occur, or they can occur at the end points of the interval. Relative extrema, by definition, can not occur at endpoints.

If $f(x)$ has a relative extremum at $(c, f(c))$, then c is a **critical number**, which means either:

$f'(c) = 0$ or f is not differentiable at c $(f'(c) =$ DNE$)$

If $f(x)$ has a critical number at c, then $f(x)$ does not necessarily have a relative extremum at c.

$f(x) = (x-1)^3$

$f(x) = \sqrt[3]{x}$

To find relative extrema, we will find the critical numbers, but we will need more information before determining whether or not there is a relative extremum at c.

For now we will just find critical numbers, but draw no conclusions regarding relative extrema.

Examples: Find critical numbers.

1. $f(x) = x\sqrt{4 - x^2}$

2. $f(x) = \begin{cases} x^2 + 4x & \text{if } x < -1 \\ 1 - x^2 & \text{if } x \geq -1 \end{cases}$

Guidelines for finding extrema for a continuous function on a closed interval $[a, b]$.

1. Find the critical numbers of $f(x)$ on (a, b).
2. Evaluate $f(x)$ at each critical number in (a, b).
3. Evaluate $f(x)$ at each endpoint of $[a, b]$.
4. The least of these values is the minimum. The greatest is the maximum.

Find absolute extrema:

3. $f(x) = x^3 - 12x$ on $[0, 4]$

4. $f(x) = \dfrac{x^2}{x^2 + 3}$ on $[-1, 1]$

5. $f(x) = x^{\frac{6}{5}} - 12x^{\frac{1}{5}}$ on $[-1, 32]$

6. $f(x) = \begin{cases} 2 - x^2 & \text{if } -2 \le x \le 3 \\ x^2 + 5x - 31 & \text{if } 3 < x < 4 \end{cases}$

7. A cardboard box manufacturer wishes to make open boxes from rectangular pieces of cardboard with dimensions 10 in. by 17 in. by cutting equal squares from the four corners and turning up the sides. Find the length of the side of the cut-out square so that the box has the largest possible volume.

8. $f(x) = \sin^2 x - x$ on $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$