

# Areas and Volumes of Revolution Lab

**Purpose:** To geometrically explore the application of the definite integral of a function

**Introduction:** In this activity you will explore the geometric applications of the definite integral of a function. We will also use integration to calculate area and volume with a variety of cross-sections. We will also use Maple to plot the graphs of area and volume. **You will need to use the Maple Worksheet created to go with this activity.**

## **Exercise 1: Volumes of Revolution**

For this exercise, consider the following functions on the interval  $[0,1]$ :

$$f(x) = 2 - x^2 \quad \text{and} \quad g(x) = 1 - x$$

- a) Graph the **area** between  $f(x)$  and  $g(x)$  on  $[0,1]$  (use the **shadearea** procedure and include a printout). Set-up an **integral** and calculate by hand this *area*.

b) Graph the **volume** formed by rotating the area between  $f(x)$  and  $g(x)$  on  $[0,1]$  around the  $x$ -axis (use the **circlevolume** procedure and include printout). Set-up an **integral** and calculate by hand this *volume*.

c) Graph the **volume** formed by rotating the area between  $f(x)$  and  $g(x)$  on  $[0,1]$  around the  $y$ -axis (use the **cylindervolume** procedure and include printout). Set-up an **integral** and calculate by hand this *volume*.

## **Exercise 2: Volume with Square Cross-Sections**

For this exercise, consider the following functions on the interval  $[\frac{\pi}{4}, \frac{5\pi}{4}]$ :

$$f(x) = \sin x + 1 \quad \text{and} \quad g(x) = \cos x + 1$$

- a) Graph the **area** between  $f(x)$  and  $g(x)$  on  $[\frac{\pi}{4}, \frac{5\pi}{4}]$  (include printout). Set-up an **integral** and calculate by hand this *area*.
- b) Graph the **volume** formed by square cross-sections with base the difference between  $f(x)$  and  $g(x)$  on  $[\frac{\pi}{4}, \frac{5\pi}{4}]$  (use the **squarevolume** procedure and include printout). Set-up an integral and calculate by hand this *volume*.

- c) Graph the **volume** formed by equilateral triangle cross-sections with base the difference between  $f(x)$  and  $g(x)$  on  $[\frac{\pi}{4}, \frac{5\pi}{4}]$  (use the **squarevolume** procedure and include printout). Set-up an integral and calculate by hand this *volume*.

**Supplement**  
**Finite and Infinite Areas and Volumes**

For this exercise, consider the following functions on the interval  $[1, \infty)$ :

$$f(x) = \frac{1}{\sqrt{1+x^2}} \quad \text{and} \quad g(x) = \frac{1}{1+x^2}$$

- a) Show that the **area** between  $f(x)$  and the  $x$  – axis on the interval  $[1, \infty)$  is *infinite*.

Now find the **volume** formed by rotating this infinite area around the  $x$  – axis.  
(Set-up and evaluate integral)

- b) Show that the **area** between  $g(x)$  and the  $x$  – axis on the interval  $[1, \infty)$  is *finite*.

Now find the **volume** formed by rotating this finite area around the  $y$  – axis.  
(Set-up and evaluate integral)

- c) What can you conclude about the *finiteness* or *infiniteness* of **area** as it relates to **volume**, and vice-versa?