

## Differential Equations: Growth and Decay

Solve the differential equation  $y' = \frac{2x}{y}$ .

In many applications, the rate of change of a variable  $y$  is proportional to the value of  $y$ . If  $y$  is a function of time  $t$ , the proportion is written as

$$\frac{dy}{dt} = ky$$

In general, if  $y$  is a differentiable function of  $t$  such that  $y > 0$  and  $y' = ky$ , for some constant  $k$ , then

$$y = Ce^{kt},$$

where  $C$  is the initial value of  $y$ , and  $k$  is the constant of proportionality. Exponential growth occurs when  $k > 0$ , and exponential decay occurs when  $k < 0$ .

1. Suppose that 10 grams of the plutonium isotope Pu-239 was released in the Chernobyl nuclear accident. How long will it take for the 10 grams to decay to 1 gram?  
The half-life of Pu-239 is 24,100 years.



Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding air.

3. A container of hot liquid is placed in a freezer that is kept at a constant temperature of  $20^{\circ}\text{F}$ . The initial temperature of the liquid is  $160^{\circ}\text{F}$ . After 5 minutes, the liquid's temperature is  $60^{\circ}\text{F}$ . How much longer will it take for the liquid's temperature to decrease to  $30^{\circ}\text{F}$ ?