The Geometry of Derivatives Lab

Purpose: To geometrically and graphically explore the relationship between a function and its derivatives.

Introduction: In this activity you will explore the geometric relationship between the graphs of a function and its first and second derivatives. We will compare patterns in the behavior of the first and second derivative with the slope and concavity of the original function. We will also use the behavior of the first and second derivatives to determine the geometry of the function, and use the graphic behavior of a function to determine the differentiability of a function.

Exercise 1: Geometric Relationship Between a Function and its Derivatives

For this exercise, consider the following function:

$$f(x) = \frac{16+4x^3-x^4}{8}$$

a) Find $f'(x)$ and $f''(x)$ using the limit definition of the derivative on Derive or Maple. Include a well-labeled printout of your work.

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ and $f''(x) = \lim_{h\to 0} \frac{f'(x+h)-f'(x)}{h}$

$f'(x) =$

$f''(x) =$

b) Graph the function and its derivatives together and answer the following questions: (include a printout of your graphs and label $f(x)$, $f'(x)$ and $f''(x)$)

On which intervals is the function $f(x)$:

Increasing	**Decreasing**	**Horizontal**

On which intervals is the first derivative $f'(x)$:

Positive	**Negative**	**Zero**

On which intervals is the function $f(x)$:

Concave Up	**Concave Down**	**Inflection Point**

On which intervals is the second derivative $f''(x)$:

Positive	**Negative**	**Zero**

On which intervals is the first derivative $f'(x)$:

Increasing	**Decreasing**	**Horizontal**

Are there any similarities in your answers? Can you explain them?

What can you conclude about the behavior of the *first derivative* and the geometry of the function?

What can you conclude about the behavior of the *second derivative* and the geometry of the function?

What can you conclude about the behavior of the *second derivative* and the geometry of the *first derivative*?

Exercise 2: **Interpreting the Geometry of a Function from its Derivative**

For this exercise, consider the following *first derivative*:

$$g'(x) = \frac{4x}{1+x^2}$$

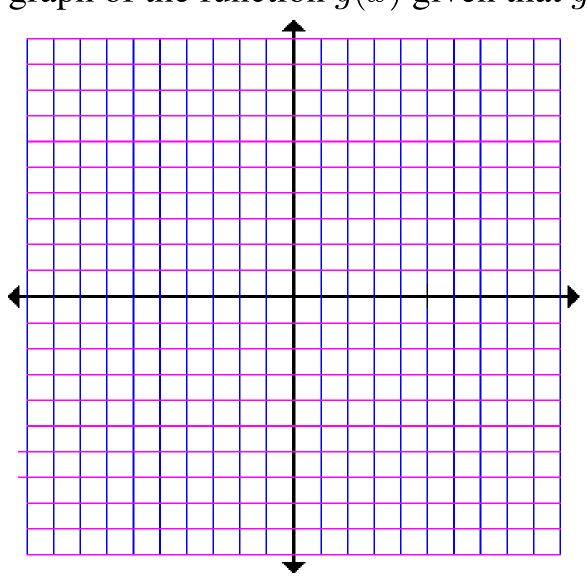
a) Graph this derivative and answer the following questions about the function $g(x)$. (Include a printout of the derivative, $g'(x)$, labeled)

b) On which intervals is the original function $g(x)$:

Increasing	**Decreasing**	**Horizontal**

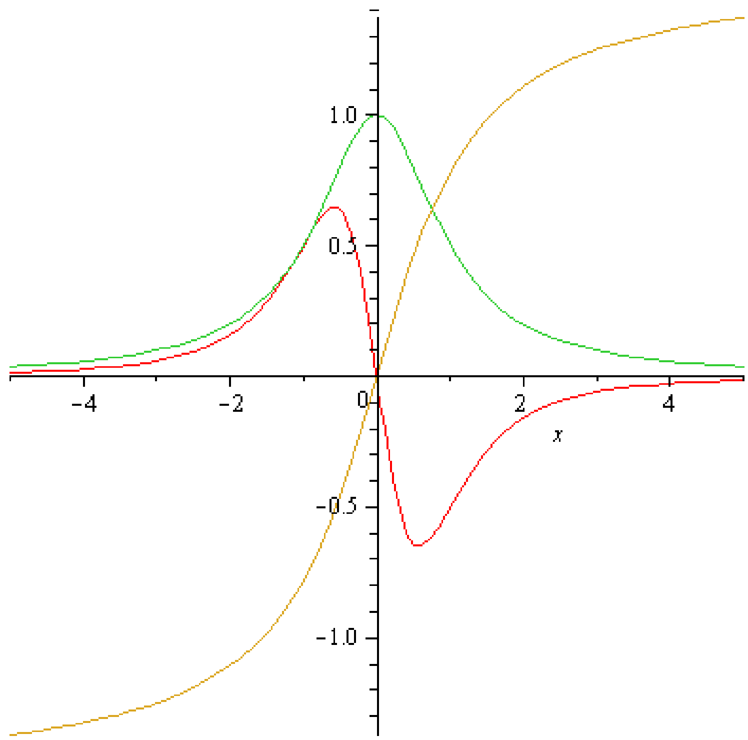
Concave Up	**Concave Down**	**Inflection Point**

d) Using the geometry of the function $g(x)$ that you have concluded from its derivative, by hand sketch a graph of the function $g(x)$ given that $g(0) = 0$.



<u>Exercise 3:</u> Distinguishing Between a Function and its Derivative

For this exercise, the graphs of a function $f(x)$ and its first and second derivative $f'(x)$ and $f''(x)$ appear (in no particular order) in the following plot:



Determine which graph is which and explain your reasoning in detail.

Exercise 4: **Determining Differentiability**

For this exercise, consider the following functions, use Derive or graphing calculator to zoom in and determine whether or not the function $f(x)$ is **differentiable** at a point a; if not, why not; if so, then find the derivative $f'(a)$:
(Include printouts of your labeled graphs or copy by hand below)

$f(x)$ when $x \neq a$	a	$f(a)$	Is $f(x)$ differentiable at a? Why?	$f'(a)$
$2 - \sqrt{x^2 - 2x + 1}$	1	2		
$4 - \sqrt[3]{x^2 + 6x + 9}$	-3	4		
$\frac{1-\cos(x)}{x^2}$	0	0.5		
$\sqrt[3]{x}\cos(\frac{1}{\sqrt[3]{x}})$	0	0		
$\frac{e^{-\frac{1}{x^2}}}{x^2}$	0	0		