Hyperbolic Functions

Hyperbolic Funtions arose from comparison of the area of a semicircular region with the area of a region under a hyperbola:

$$\int_{-1}^{1}\sqrt{1-x^2}\,dx = \frac{1}{2}[x\sqrt{1-x^2} + \text{arcsin}x]$$

$$\int_{-1}^{1}\sqrt{1+x^2}\,dx = \frac{1}{2}[x\sqrt{1+x^2} + \text{sinh}x]$$

Definitions:

$sinhx = \frac{e^x - e^{-x}}{2}$

$coshx = \frac{e^x + e^{-x}}{2}$

$tanh = \frac{sinhx}{coshx}$

$cschx = \frac{1}{sinh}, x \neq 0$

$sechx = \frac{1}{coshx}$

$coth = \frac{1}{tanh}, x \neq 0$

Identities:

$cosh^2x - sinh^2x = 1$

Proof:

$2sinhxcoshx = sinh2x$

Proof:

Derivatives and Integrals of Hyperbolic Functions:

$\frac{d}{dx}[sinhu] = (coshu)u'$

$\frac{d}{dx}[coshu] = (sinhu)u'$

$\frac{d}{dx}[tanhu] = (sech^2u)u'$

$\int coshudu =$

$\int sinhudu =$

$\int sech^2udu =$

1. $\frac{d}{dx}[xsinhx - coshx] =$

2. $\frac{d}{dx}[ln(coshx)] =$

3. $\int cosh^2(x-1)sinh(x-1)dx =$

4. $\int \frac{cosh\sqrt{x}}{\sqrt{x}}dx =$

5. Power cables are suspended between two towers, forming the caternary shown below. The equation for this caternary is

$$y = acosh\frac{x}{a}$$

The distance between the two towers is $2b$. Find the slope of the caternary at the point where the cable meets the right-hand tower.