

Implicit Differentiation

Implicit vs. Explicit form of an equation

$$3x - y - 2 = 0$$

$$y =$$

$$xy - 1 = 0$$

$$y =$$

$$x^2 + y^2 = 36$$

$$y =$$

$$2x^2 - 3y^3 + 2y = 2$$

$$y =$$

Recall that $\frac{d}{dx}(x^2) = 2x$ $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$ where y is a function of x .

The process of implicit differentiation is used to find $\frac{dy}{dx}$ in situations where y is not explicitly defined as a function of x .

If we wanted to find the slope of the tangent line on the curve $2x^2 - 3y^3 + 2y = 2$ at the point $(1, 0)$, we could use implicit differentiation to find the slope of the tangent line. In using implicit differentiation we need to realize that in differentiating functions involving x alone we differentiate as usual. However in differentiating functions involving y , we need to realize that y is a function of x and therefore the chain rule needs to be used.

The implicit differentiation process is as follows:

1. Differentiate both sides of the equation with respect to x
2. Collect all terms involving $\frac{dy}{dx}$ on one side of the equation and all other terms on the other side.
3. Factor $\frac{dy}{dx}$ out of all terms on the side of the equation involving $\frac{dy}{dx}$.
4. Divide both sides of the equation by the coefficient of $\frac{dy}{dx}$.

Examples:

1. Find the equation of the tangent line to $2x^2 - 3y^3 + 2y = 2$ at the point $(1, 0)$.

Find y' in each of the following:

2. $x^2y + y^2x = -2$

3. $\sqrt{xy} = x - 2y$

4. $2\sin x \cos y = 1$

5. $x = \sec \frac{1}{y}$

6. $x \cos y = 1$ Find $\frac{dy}{dx}$ at $\left(2, \frac{\pi}{3}\right)$

7. Find the equation of the tangent line to $y^2 = \frac{x-1}{x^2+1}$ at $\left(2, \frac{\sqrt{5}}{5}\right)$

8. Find the points at which the graph of the equation has a vertical or horizontal tangent line.

$$4x^2 + y^2 - 8x + 4y + 4 = 0$$

9. Find y'' if $x^2 + 25y^2 = 100$.

10. $x^2 + y^2 = 9$ Find equations for the tangent line and the normal line at $(2, \sqrt{5})$.

On occasion it is useful to use logarithms to help differentiate nonlogarithmic functions. This is called **logarithmic differentiation**.

11. Differentiate $y = \frac{x^3\sqrt{1-x}}{(2x+1)^2}$

12. Find $\frac{dy}{dx}$: $y = x^{x-1}$

13. Find the derivative of $y = (\sin x)^{\frac{1}{x}}$