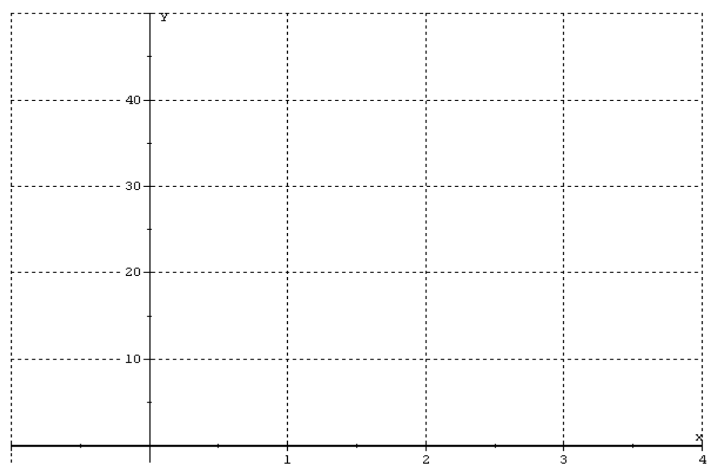
Infinite Limits

$$f(x) = \frac{3}{(x-2)^2}$$

x	$f(x) = \frac{3}{(x-2)^2}$	x	$f(x) = \frac{3}{(x-2)^2}$
1		3	
1.5		2.5	
$1\,\frac{2}{3}$		$2\,\frac{1}{3}$	
1.75		2.25	
1.9		2.1	
1.99		2.01	
1.999		2.001	

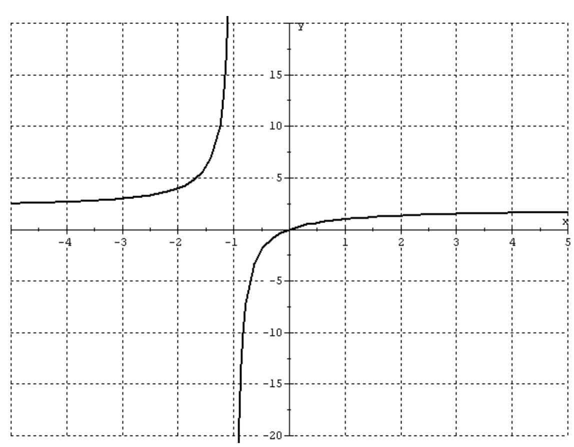


<u>Definition</u>: Let f be a function that is defined at every number in some open interval containing a, except possibly at a itself. As x approaches a, $f(x)$ increases without bound, which is written $\lim_{x\to a} f(x) = +\infty$ if for any number $N > 0$, however large, $\exists\, \delta > 0$ such that if

Let $f(x) = -\dfrac{3}{(x-2)^2}$ $\qquad \lim_{x \to 2} -\dfrac{3}{(x-2)^2} =$

<u>Definition</u>: Let f be a function that is defined at every number in some open interval containing a, except possibly at a itself. As x approaches a, $f(x)$ decreases without bound, which is written $\lim_{x \to a} f(x) = -\infty$ if for any number $N < 0$, $\exists\, \delta > 0$, such that if

$f(x) = \dfrac{2x}{x+1}$



$\lim_{x \to -1^-} f(x) =$ $\qquad \lim_{x \to -1^+} f(x) =$

Definition: The line $x = a$ is said to be a vertical asymptote of the graph of the function f if at least one of the following statements is true:

a. $\lim_{x \to a^+} f(x) = +\infty$

b. $\lim_{x \to a^+} f(x) = -\infty$

c. $\lim_{x \to a^-} f(x) = +\infty$

d. $\lim_{x \to a^-} f(x) = -\infty$

$\lim_{x \to a} f(x) = -\infty$

$\lim_{x \to a} f(x) = +\infty$

Infinite Limit Theorem and Vertical Asymptotes

If a is any real number and if $\lim_{x\to c} g(x) = 0$ and $\lim_{x\to c} f(x) = a \neq 0$, then:

a. If $a > 0$ and $g(x) \to 0$ through positive values of $g(x)$,

$\lim_{x\to c} \frac{f(x)}{g(x)} =$ and f has a vertical asymptote at $x = c$.

b. If $a > 0$ and $g(x) \to 0$ through negative values of $g(x)$,

$\lim_{x\to c} \frac{f(x)}{g(x)} =$ and f has a vertical asymptote at $x = c$.

c. If $a < 0$ and $g(x) \to 0$ through positive values of $g(x)$,

$\lim_{x\to c} \frac{f(x)}{g(x)} =$ and f has a vertical asymptote at $x = c$.

d. If $a < 0$ and $g(x) \to 0$ through negative values of $g(x)$,

$\lim_{x\to c} \frac{f(x)}{g(x)} =$ and f has a vertical asymptote at $x = c$.

The theorem is also valid if $x \to c$ is replaced by $x \to c^+$ or $x \to c^-$.

Also, it is still the case that if $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c} g(x) = 0$, that we have an indeterminate form when evaluating
$\lim_{x\to c} \frac{f(x)}{g(x)}$.

THEOREM **Properties of Infinite Limits**

Let c and L be real numbers and let f and g be functions such that

$$\lim_{x\to c} f(x) = \infty \quad \text{and} \quad \lim_{x\to c} g(x) = L.$$

1. Sum or difference: $\lim_{x\to c}[f(x) \pm g(x)] =$
2. Product: $\lim_{x\to c}[f(x)g(x)] =$, $L > 0$

   $\lim_{x\to c}[f(x)g(x)] =$, $L < 0$
3. Quotient: $\lim_{x\to c} \dfrac{g(x)}{f(x)} =$

Similar properties hold for one-sided limits and for functions for which the limit of $f(x)$ as x approaches c is $-\infty$.

Find each of the following limits and sketch a local graph illustrating the limit you have found.

$$\lim_{x\to 2^+} \frac{x+3}{x-2}$$

$$\lim_{x\to 2^+} \frac{x-5}{x-2}$$

$$\lim_{x\to 3^-} \frac{1-x}{x-3}$$

$$\lim_{x \to 3^-} \frac{2x - 1}{x - 3}$$

$$\lim_{x \to 2^+} \frac{x + 1}{x^2 - 8x + 12}$$

$$\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$$

$$\lim_{x \to 5^+} \frac{x^2 - 9}{x^2 - 3x - 10}$$

$$\lim_{x \to 5^-} \frac{x^2 - 9}{x^2 - 3x - 10}$$

$$\lim_{x \to -2^-} \frac{x^2 - 9}{x^2 - 3x - 10}$$

$$\lim_{x \to -2^+} \frac{x^2 - 9}{x^2 - 3x - 10}$$

$$\lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$$

$$\lim_{x \to 0^-} \left(\frac{1}{x^2} - \frac{1}{x} \right)$$

$$\lim_{x \to 0^+} \left(\frac{1}{x^2} - \frac{1}{x} \right)$$

$$\lim_{x \to 2^-} \frac{[[x]] - x}{2 - x}$$

$$\lim_{x \to 2^+} \left(\frac{3}{x^2 - 5x + 6} - \frac{1}{x - 2} \right)$$

$$\lim_{x \to 2^-} \left(\frac{3}{x^2 - 5x + 6} - \frac{1}{x - 2} \right)$$

<u>True or false</u>: If $p(x)$ is a polynomial, then the function given by $f(x) = \dfrac{p(x)}{x - 1}$ has a vertical asymptote at $x = 1$.