

Integration of the Natural Logarithm

We know that $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$.

Therefore $\int \frac{1}{u} du = \ln|u| + C$ or

equivalently $\int \frac{du}{u} = \ln|u| + C$ and $\int \frac{1}{x} dx = \ln|x| + C$.

Derive antiderivatives for the rest of the trigonometric functions.

1. $\int \tan x dx$

2. $\int \cot x dx$

3. $\int \sec x dx$

4. $\int \csc x dx$

Guidelines for Integration

1. Memorize the list of 12 integral formulas we have - power rule, log rule, and 10 trigonometric rules.
2. Determine whether the integrand fits one of the 12 formulas directly or can be altered by multiplying numerator and denominator by a constant so that it will fit one of the 12 formulas.
3. Find an integration formula that resembles all or part of the integrand and, by trial and error, find a choice of u that will make the integrand conform to one of our 12 formulas.
4. If you cannot find a substitution for u , try altering the integrand by applying a trigonometric identity, multiplying numerator and denominator by the same quantity, or by adding and subtracting the same quantity to the integrand.
5. Use DERIVE.
6. You can check your answer by differentiating.

Examples:

1. $\int \frac{dx}{6x + 1}$

2. $\int \frac{x^2}{3 - x^3} dx$

3.
$$\int_e^{e^2} \frac{1}{x \ln(x^2)} dx$$

4.
$$\int \frac{1}{x^{\frac{2}{3}} (1 + x^{\frac{1}{3}})} dx$$

5.
$$\int \frac{\sin x}{1 + \cos x} dx$$

6.
$$\int \frac{\sec(\ln x)}{x} dx$$

$$7. \int \frac{x-1}{x+1} dx$$

$$8. \int \frac{\sec^2 t}{\tan t + 1} dt$$

$$9. \int \sqrt{1 + \tan^2 x} dx$$

$$10. \int \frac{x(x-2)}{(x-1)^3} dx$$

11. Find the area bounded by $f(x) = \frac{2x^2 - 14x + 24}{x - 1}$ and the x -axis.