

L'Hopital's Rule

Indeterminate Forms: $\frac{0}{0}$ $\frac{\infty}{\infty}$ $\infty - \infty$ $0 \cdot \infty$ 1^∞ ∞^0 0^0

These are indeterminate because they don't guarantee that a limit exists, nor do they indicate what the limit is, if one does exist.

L'Hopital's Rule: Let f and g be functions that are differentiable on an open interval (a, b) containing c , except possibly at c itself. Assume $g'(x) \neq 0$ for all x in (a, b) , except possibly at c itself. If the limit of $f(x)/g(x)$ as x approaches c produces the indeterminate form $\frac{0}{0}$ or $\pm \frac{\infty}{\infty}$, then

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is infinite).

1. $\lim_{x \rightarrow 0} \frac{e^{3x} - x - 1}{x}$

2. $\lim_{x \rightarrow \infty} \frac{x}{\ln x}$

3. $\lim_{x \rightarrow \infty} \frac{x^2}{e^x}$

4. $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$

5.
$$\lim_{x \rightarrow \infty} e^{-x} \sqrt{x}$$

6.
$$\lim_{x \rightarrow \infty} (1 + x)^{\frac{1}{x}}$$

7.
$$\lim_{x \rightarrow 0^+} (\sin x)^x$$

8.
$$\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$$

$$9. \quad \lim_{x \rightarrow 0^+} \sqrt{x} \ln x$$

$$10. \quad \lim_{x \rightarrow 0^+} x^{\frac{\ln 2}{1 + \ln x}}$$

$$11. \quad \lim_{x \rightarrow 0} (x + 1)^{\frac{\ln 2}{x}}$$