

## L'Hopital's Rule

**Indeterminate Forms:**  $\frac{0}{0}$   $\frac{\infty}{\infty}$   $\infty - \infty$   $0 \cdot \infty$   $1^\infty$   $\infty^0$   $0^0$

These are indeterminate because they don't guarantee that a limit exists, nor do they indicate what the limit is, if one does exist.

**L'Hopital's Rule:** Let  $f$  and  $g$  be functions that are differentiable on an open interval  $(a, b)$  containing  $c$ , except possibly at  $c$  itself. Assume  $g'(x) \neq 0$  for all  $x$  in  $(a, b)$ , except possibly at  $c$  itself. If the limit of  $f(x)/g(x)$  as  $x$  approaches  $c$  produces the indeterminate form  $\frac{0}{0}$  or  $\pm \frac{\infty}{\infty}$ , then

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is infinite).

1.  $\lim_{x \rightarrow 0} \frac{e^{3x} - x - 1}{x}$

2.  $\lim_{x \rightarrow \infty} \frac{x}{\ln x}$

3.  $\lim_{x \rightarrow \infty} \frac{x^2}{e^x}$

4.  $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$

5. 
$$\lim_{x \rightarrow \infty} e^{-x} \sqrt{x}$$

6. 
$$\lim_{x \rightarrow \infty} (1 + x)^{\frac{1}{x}}$$

7. 
$$\lim_{x \rightarrow 0^+} (\sin x)^x$$

8. 
$$\lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right)$$

$$9. \quad \lim_{x \rightarrow 0^+} \sqrt{x} \ln x$$

$$10. \quad \lim_{x \rightarrow 0^+} x^{\frac{\ln 2}{1 + \ln x}}$$

$$11. \quad \lim_{x \rightarrow 0} (x + 1)^{\frac{\ln 2}{x}}$$