Limits at Infinity

Let $f(x) = \frac{3x^2}{x^2+1}$

x	$-\infty$	-100	-10	-1	0	1	10	100	∞
$f(x)$	3	2.997	2.97	1.5	0	1.5	2.97	2.997	3

Horizontal Asymptotes:

The line $y = L$ is a horizontal asymptote of the graph f if

$$\lim_{x \to -\infty} f(x) = L$$

or

$$\lim_{x \to \infty} f(x) = L$$

Note: The graph of a function of x can have at most two horizontal asymptotes - one to the right and one to the left.

Limits at infinity have many of the same properties of limits we've already discussed. Some other properties you should know:

$\lim_{x \to 0^+} \frac{1}{x} =$ $\lim_{x \to 0^-} \frac{1}{x} =$

$\lim_{x \to \infty} \frac{1}{x} =$ $\lim_{x \to -\infty} \frac{1}{x} =$

In general, $\lim_{x \to \pm\infty} \frac{c}{ax^n} = 0$

Example: $\lim_{x \to \infty} \frac{3x-1}{2x-5} =$

$\lim_{x \to \infty} \frac{2x^2-5x+7}{7x^2-x+4} =$ $\lim_{x \to \infty} \frac{2x^3-5x+7}{7x^2-x+4} =$ $\lim_{x \to \infty} \frac{2x^3-5x+7}{7x^6-x+4} =$

Guidelines for Finding Limits at Infinity:

1. If the degree of the numerator is less than the degree of the denominator, then the limit of the rational function is 0.
2. If the degree of the numerator is equal to the degree of the denominator, then the limit of the rational function is the ratio of the leading coefficients.
3. If the degree of the numerator is greater than the degree of the denominator, then the limit of the rational function does not exist.

Horizontal Asymptotes occur when the degree of the numerator is less than or equal to the degree of the denominator.

Slant or Oblique Asymptotes occur when the degree of the numerator is one more than the degree of the denominator.

Find the asymptotes of:

$f(x) = \frac{x^2-1}{x^2-x-6}$

$f(x) = \frac{x+2}{x^2+2x-15}$

Find the horizontal asymptotes of the logisitic function $f(x) = \frac{1}{1+e^{-x}}$

Find the limit $\lim_{x \to -\infty} \frac{-3x}{\sqrt{x^2+x}}$

Find the limits:

$\lim_{x \to \infty} cosx$

$\lim_{x \to \infty} \frac{sinx}{x}$

Summary:

$$\lim_{x\to\infty}\frac{sinx}{x}=$$

$$\lim_{x\to\infty}\frac{x}{sinx}=$$

$$\lim_{x\to 0}\frac{sinx}{x}=$$

$$\lim_{x\to 0}\frac{x}{sinx}=$$

$$\lim_{t\to\infty}\frac{t}{sin4t}$$

$$\lim_{x\to\infty}\left[\frac{5}{2}+ln\left(\frac{x^2+1}{x^2}\right)\right]$$

$$\lim_{x\to\infty}\frac{3x^2-6x}{x+1}$$

$$\lim_{x\to\infty}xtan\frac{1}{x}$$