Optimization Problems

<u>Strategy</u>

1. **Read the problem carefully. Make a drawing if necessary.**
2. **Label all variables, noting what varies and what stays fixed.**
3. **Translate the problem into an <u>objective function</u> (whatever quantity needs to be maximized or minimized). You may also need a <u>constraint equation</u> (whatever quantity stays fixed.)**
4. **Use the constraint equation to solve for a variable and rewrite the objective equation only in terms of one variable.**
5. **Maximize or minimize the objective by setting the derivative to zero or DNE.**
6. **Verify that a max or min has been achieved.**

1. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be maximum?

2. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

3. A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are each $1\frac{1}{2}$ inches. The margins on each side are 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

4. An open box is to be made from a rectangle piece of material (2 ft by 3 ft) by cutting equal squares from each corner and turning up the sides. Find the dimensions of the box of maximum volume.

5. Find two positive numbers such that the product of the numbers is 192 and the sum of the first plus three times the second is a minimum.

6. Find the point on the circle $x^2 + y^2 = 9$ that is closest to the point $(4, 5)$.

7. If a closed tin can of volume 60 in^3 is to be in the form of a right-circular cylinder, find the base radius of the can if the least amount of tin is to be used in its manufacture.

8. A Norman window consists of a rectangle surmounted by a semicircle. If the perimeter of a Norman window is to be 32 feet, determine what should be the radius of the semicircle and the height of the rectangle such that the window will admit the most light.

9. The cross sections of an irrigation canal are isosceles trapezoids of which three of the sides are 8 feet long. Determine the angle of elevation θ of the sides so that the area of the cross section is maximum .