Pattern Recognition and U-Substitution

Recall functions we know how to integrate.

When integrating composite functions we will use an appropriate substitution to get our integrand into one of these forms.

Recall the Chain Rule: $\frac{d}{dx}(F(g(x)) = F'(g(x))g'(x) \qquad \Rightarrow$

$$\int F'(g(x))g'(x)\,dx = F(g(x)) + C$$

This may be written in a different form if we let $F' = f$, $u = g(x)$, and $du = g'(x)$

$$\int f(u)\,du = F(u) + C \quad \text{or} \quad \int f(g(x))g'(x)\,dx = F(g(x)) + C$$

1. What would we need in each of the following for the integrand to be in the above form?

$\int \left(x^4 + 2x\right)^5 (\qquad\qquad)\,dx$

$\int \sqrt{\sin x}\ (\qquad\qquad)\,dx$

$\int \cos\left(\sin x^2\right)(\qquad\qquad)\,dx$

$\int (\tan x + 2)^5 (\qquad\qquad)\,dx$

2. The integrand in each of the following integrals fits the pattern $f(g(x))g'(x)$. Identify the pattern and use the above result to evaluate the integral. This is called **Pattern Recognition**.

a) $\int 2x\left(x^2-3\right)^4 dx$

b) $\int \frac{3x^2}{\left(x^3+1\right)^7} dx$

c) $\int \sec^2 x \tan x \, dx$

3. The following integrals are similar to the previous integrals. Multiply and divide by the appropriate constant to evaluate each of the integrals.

a) $\int x\left(x^2-3\right)^4 dx$

b) $\int \frac{x^2}{\left(x^3+1\right)^7} dx$

U-Substitution is an alternative to pattern recognition. It is useful when it is not as obvious what pattern to use.

4. Complete the following table:

	$u = g(x)$	$du = g'(x)dx$
$\int x^2 \sqrt{x^3+5}\, dx$		
$\int \sec 2x \tan 2x \, dx$		
$\int \frac{\cos x}{\sin^2 x}\, dx$		

Evaluate each of the following:

5. $\displaystyle\int 2\pi x\left(8 - x^{\frac{3}{2}}\right)dx$

6. $\displaystyle\int_0^4 x\,\sqrt{2x+1}\,dx$

7. $\displaystyle\int_1^2 (x-1)\sqrt{2-x}\,dx$

8. $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$

9. $\int \sin x \cos x \, dx$

10. $\int \tan x \sec^2 x \, dx$

Pattern Recognition vs. U-substitution (change in variables)

11. $\int x\left(x^2+5\right)^3 dx$

12. $\int \cos 3x\, dx$

Generalized Power Rule: $\int [g(x)]^n g'(x)\, dx =$

Solve the differential equation. $\dfrac{dy}{dx} = \dfrac{x-4}{\sqrt{x^2-8x+1}}$

13. If $f(x)$ is an even function evaluate $\int_{-a}^{a} f(x)\,dx$

14. If $f(x)$ is an odd function evaluate $\int_{-a}^{a} f(x)\,dx$

15. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos x\,dx$

16. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \cos x\,dx$