The First Derivative Test

The first and second derivatives of a function are tools of Calculus that give information about the shape of a graph that is useful for finding maximums, minimums and graphing.

Increasing: A graph is increasing if it rises from left to right.
If $a < b$, then $f(a) < f(b)$.

Decreasing: A graph is decreasing if it falls from left to right.
If $a < b$, then $f(a) > f(b)$.

Theorem:
If $f'(x) > 0$ for all x in an interval I, then f is **increasing** over I.
If $f'(x) < 0$ for all x in an interval I, then f is **decreasing** over I.

A function is **strictly monotonic** on an interval if it is either increasing on the entire interval or decreasing on the entire interval.

Note: A function can only change from increasing to decreasing at critical points.

1. Find the intervals of increase and decrease for the function

$$f(x) = 12 + 9x - 3x^2 - x^3$$

Recall that critical points are candidates for relative extrema, but they may not necessarily be relative maxs or mins. So we will need to use the First Derivative Test to determine where relative extrema actually occur and what kind they are.

Relative Minimum Relative Maximum No relative max or min

Theorem: The First Derivative Test for Relative Extrema
For any continuous function f that has exactly one critical point c in an open interval (a, b) :

1) f has a relative minimum at c if: f' changes from negative to positive

2) f has a relative maximum at c if: f' changes from positive to negative

3) f has no relative max or relative min at c if: f' doesn't change signs

2. Graph the function $f(x) = 12 + 9x - 3x^2 - x^3$

3. Let $f(x) = \frac{5}{x^2+1}$. Find the relative extrema and graph the function.

4. Let $f(x) = (x+3)^{\frac{2}{3}} - 5$. Find the relative extrema and graph the function.

5. Let $f(x) = \sin x \cos x$. Find the intervals of increase and decrease and the relative extrema on $[0, 2\pi]$.