Cross Sectional Areas and Volumes:

Using the disc method we can find the volume of a solid with a circular cross section. Recall that the circular cross section has an area of πr^2. We can find the volume of the region by summing the areas of all of the circular cross sections using integration. This method can be generalized to find the volume of solids of any shape, as long as we can find a formula for the area of the arbitrary cross section.

If the cross sections of the area are taken perpendicular to the x-axis the volume is given by the formula $\int_a^b A(x)dx$.

If the cross sections of the area are taken perpendicular to the y-axis, the volume is given by the formula $\int_c^d A(y)dy$.

1. Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 4$, with the indicated cross sections taken perpendicular to the x-axis.

a. Squares

b. Semicircles

c. Rectangles of height 2

2. Find the volume of the solid whose base is the area bounded by the lines $f(x) = 2 - x$, $g(x) = x - 2$, and $x = 0$, whose cross sections perpendicular to the x-axis are squares.

3. What would change if the cross section perpendicular to the x-axis was:

a. an equilateral triangle?

b. semicircle?

4. Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the indicated cross sections taken perpendicular to the x-axis.

a. Squares

b. Rectangles of height 1