Cylindrical Shells

Find the volume of the solid formed by revolving the region bounded by the graphs $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.

In this section we will look at alternate way to calculate the volume of a solid of revolution. Rather than taking the cross section perpendicular to the axis of rotation we will take the cross section parallel to the axis of rotation.

This is called the **cylindrical shell** method:

In the cylindrical shell method, always take the cross section parallel to the axis of revolution.

Examples:

1. Find the volume of a solid of revolution of the region formed by $y = 1 - x$, x-axis, and the y-axis is revolved about

a) y-axis

b) $x = 4$

c) $x = -1$

2. Find the volume of a solid of revolution of the region in the first quadrant formed by $y = x^2 + 4, y = 8$, and the y-axis is revolved about

a) y-axis

b) $x = 4$

c) $y = 9$

3. Find the volume of a solid of revolution of the region formed by $y = \dfrac{1}{x^2}$, $x = 1$, $x = 4$, and $y = 0$ is revolved about

a) x-axis

b) y-axis

c) $x = 4$

d) $y = 1$

4. A torus is formed revolving the region bounded by the circle $x^2 + y^2 = 1$ about the line $x = 2$. Find the volume.

Comparison of all 3 methods:

Disc/Washer Method: the cross sections are always PERPENDICULAR to the axis of revolution.

Cylindrical Shells Method: the cross sections are always PARALLEL to the axis of revolution

When do you use each method? Discs/washers are usually easier, but sometimes shells work better.

If,

1) **it is difficult to solve for y in terms of x (or vice versa) and you need to**

or

2) **the new integral you end up with looks very difficult**

or

3) **you need two integrals instead of one to solve the problem**

try another method!

Examples:

1. $y = x^2 + 1,\ x = 0,\ y = 0,\ x = 1$ revolved around y-axis

2. $y = 1 - \frac{x^2}{16},\ y = 0$ revolved around x-axis

3. $y = x^3 + x + 1,\ y = 1,\ x = 1$ revolved around $x = 2$