Disc and Washer Method

When we take a region and revolve it around a line that does not cross the interior of the region, then the resulting solid is called a **solid of revolution** and the line is the **axis of revolution**.

In each of these examples, the cross-section is always the shape of a circular disc or discs. Since the area of a circle is πr^2, then the area function we use in our volume integral is $A = \pi r^2$.

When using the disc method, always take your cross-section cut perpendicular to the axis of revolution.

Examples:

1. Find the volume of the solid of revolution formed when the region bounded by $y = 4 - x^2$, $x \geq 0, y \geq 0$ is revolved about

a) the x-axis

b) the y-axis.

2. Find the volume of the solid of revolution formed when the region bounded by $y = x^2$, $y = 0$, and $x = 1$ is revolved about the x-axis

3. The region in example 2 revolved about the line $x = 1$.

4. Find the volume of the solid of revolution formed when the region bounded by $y = x^{\frac{2}{3}},\ y = 1,\ x \geq 0$ is revolved about the y-axis.

5. The region in example 4 revolved about the line $y = 1.$

6. Find the volume of the solid of revolution formed when the region bounded by $x = -y^2 + 4y,\ y = 1,$ and the y-axis, below the line $y = 1$, revolved about the y-axis.

7. The region in example 6 revolved about the line $y = 1$.

Consider the situation if such a region as discussed so far in this section is revolved about a line that is a non-boundary.

This is generally referred to as the "washer" method. One can think of the volume created by finding the volume of the whole region minus the volume of the hole.

8. Find the volume of the solid of revolution formed when the region bounded by $y = 2x^2,\ y = 0,\ x = 2,$ about the y-axis

9. The region in example 8 about the line $y = 8$.

10. Find the volume of the solid of revolution formed when the region bounded by $y = 6 - 2x - x^2$ and the line $y = x + 6$ about the x-axis.

11. The region in example 10 about the line $y = 3$.

12. Prove that the volume of a right circular cone is $v = \frac{1}{3}\pi r^2 h$