

Exam 2 Review Sheet

Topics:

1. Find the value of an indefinite or definite integral using the techniques of integration by parts, trigonometric substitutions, partial fractions, special substitution for rational functions of sine and cosine.
2. Find the value of an indefinite or definite integral involving various powers of the trigonometric functions.
3. Use a set of tables to evaluate an indefinite or definite integral.
4. Use DERIVE or some other symbol manipulator to evaluate an indefinite or definite integral.
5. Determine whether an improper integral is convergent or divergent and if it is convergent, evaluate it.

Problems:

Evaluate the following integrals:

1. $\int x^5 e^{-x^2} dx$
2. $\int \sec^3 x dx$
3. $\int \frac{1}{x^2 - 8x + 15} dx$
4. $\int e^x \cos(e^x) dx$
5. $\int \frac{2x^3 - 4x - 8}{(x^2 - x)(x^2 + 4)} dx$
6. $\int \frac{dx}{x^2 \sqrt{16 - x^2}}$
7. $\int \frac{dx}{x^3 + 6x^2 + 10x}$
8. $\int \frac{dx}{(x^2 + 9)^2}$
9. $\int \cot^5 x dx$
10. $\int \sin^4 4x \cos^3 4x dx$
11. $\int \frac{\sqrt{x^2 - 3}}{x} dx$
12. $\int \sqrt{3 + 2x - x^2} dx$

13. $\int \frac{x}{x^2+2x+3} dx$

14. $\int \frac{1}{x^2-8x+80} dx$

Use a table of integrals to evaluate the following. Indicate any substitutions used and the number of the integral procedure used.

15. $\int \frac{x^2}{(3x-5)^2} dx$

16. $\int \frac{1}{\tan x \sqrt{4-\sin^2 x}} dx$

17. $\int \frac{\cos^3 \sqrt{x}}{\sqrt{x}} dx$

Evaluate the following improper integrals. Use proper limit notation.

18. $\int_3^\infty \frac{1}{9+x^2} dx$

19. $\int_0^2 \frac{1}{\sqrt{4-x^2}} dx$

20. $\int_3^\infty \frac{x}{9+x^2} dx$

21. $\int_{-\infty}^0 e^{3x} dx$

22. Determine the values for which $\int_e^\infty \frac{dx}{x(\ln x)^p}$ converges.
Give clear reasons for your answer.

Use the comparison test to determine whether or not the following improper integrals converge or diverge.

23. $\int_1^\infty \frac{x^3+4x}{x^4+3x^2+2} dx$

24. $\int_1^\infty \frac{\cos^2 x}{x^2+1} dx$

Answers:

1. $-\frac{(x^4+2x^2+2)}{2}e^{-x^2} + C$

2. $\frac{1}{2}(\sec x \tan x - \ln|\sec x + \tan x|) + C$

3. $-\frac{1}{2}\ln|x-3| + \frac{1}{2}\ln|x-5| + C$

4. $\sin e^x + C$

5. $2\ln|x| - 2\ln|x-1| + \ln|x^2+4| + 2\tan^{-1}\left(\frac{x}{2}\right) + C$

6. $-\frac{\sqrt{16-x^2}}{16x} + C$

7. $\frac{1}{10}\ln x - \frac{1}{20}\ln|x^2+6x+10| - \frac{3}{10}\tan^{-1}(x+3) + C$

8. $\frac{1}{54}\tan^{-1}\frac{x}{3} + \frac{x}{27(x^2+9)} + C$

9. $-\frac{(\csc x)^4}{4} + (\csc x)^2 + \ln|\sin x| + C$

10. $\frac{1}{20}(\sin 4x)^5 - \frac{1}{28}(\sin 4x)^7 + C$

11. $\sqrt{x^2-3} - \sqrt{3}\sec^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

12. $2\sin^{-1}\frac{x-1}{2} + (x-1)\left(\frac{\sqrt{3+2x-x^2}}{2}\right) + C$

13. $\frac{1}{2}\ln|x^2+2x+3| - \frac{1}{\sqrt{2}}\tan^{-1}\frac{x+1}{\sqrt{2}} + C$

14. $\frac{1}{8}\tan^{-1}\frac{x-4}{8} + C$

15. $\frac{1}{27}\left(3x - \frac{25}{3x-5} + 10\ln|3x-5|\right) + C, \#7$

16. $-\frac{1}{2}\ln\left|\frac{2+\sqrt{4-\sin^2 x}}{\sin x}\right| + C, \#42$

17. $\frac{2}{3}\cos^2\sqrt{x}\sin\sqrt{x} + \frac{4}{3}\sin\sqrt{x} + C, \#51$

18. $\frac{\pi}{12}$

19. $\frac{\pi}{2}$

20. Divergent

21. $\frac{1}{3}$

22. Convergent for $p > 1$

23. Diverges

24. Converges