Partial Fraction Decomposition

The following integral can be integrated in two different ways.

1. $$\int \frac{dx}{x^2 - 5x + 6}$$

2. $$\int \frac{dx}{x^2 - 5x + 6}$$

In general many functions can be integrated more easily using the method illustrated in example 2. This method is referred to as "Integration by means of Partial Fractions". In order to integrate using this method we need to be able to decompose rational expressions into partial fractions.

In order to decompose a partial fraction we will use the following procedure.

1. Divide if expression $\dfrac{N(x)}{D(x)}$ is improper.

2. Factor $D(x)$ into factors $(px+q)^m$ & $(ax^2+bx+c)^n$

3. For each linear factor of the form $(px+q)^m$ the partial fraction decomposition must include the following sum:

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \frac{A_3}{(px+q)^3} + \ldots + \frac{A_m}{(px+q)^m}$$

4. For each quadratic factor of the form $(ax^2+bx+c)^n$ the partial fraction decomposition must include the following sum:

$$\frac{B_1x+C_1}{(ax^2+bx+c)} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \frac{B_3x+C_3}{(ax^2+bx+c)^3} + \ldots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}$$

3. $\displaystyle\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}\,dx$

4. $\displaystyle\int \frac{2x^3 - 4x - 8}{(x^2 - x)(x^2 + 4)}\,dx$

5. $\displaystyle\int \frac{8x^3 + 13x}{\left(x^2 + 2\right)^2}\,dx$

6. $\displaystyle\int \frac{\cos x}{\sin x\,(\sin x - 1)}\,dx$

7. $\displaystyle\int \frac{x^3 - x + 3}{x^2 - x - 2}\,dx$

8. $\displaystyle\int \frac{du}{u^2 - a^2}$