Polar Coordinates and Calculus

Recall that in the Polar coordinate system the coordinates (r, θ) represent the directed distance from the pole to the point and the directed angle, counterclockwise from the polar axis to the segment from the pole to the point. A polar function would be of the form: $r = f(\theta)$.

To find the slope of the tangent line of a polar graph we will parameterize the equation. $r = f(\theta)$ (Assume f is a differentiable function of θ)
$\therefore x = r\cos\theta = f(\theta)\cos\theta$ and $y = r\sin\theta = f(\theta)\sin\theta$.
Recall also that $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

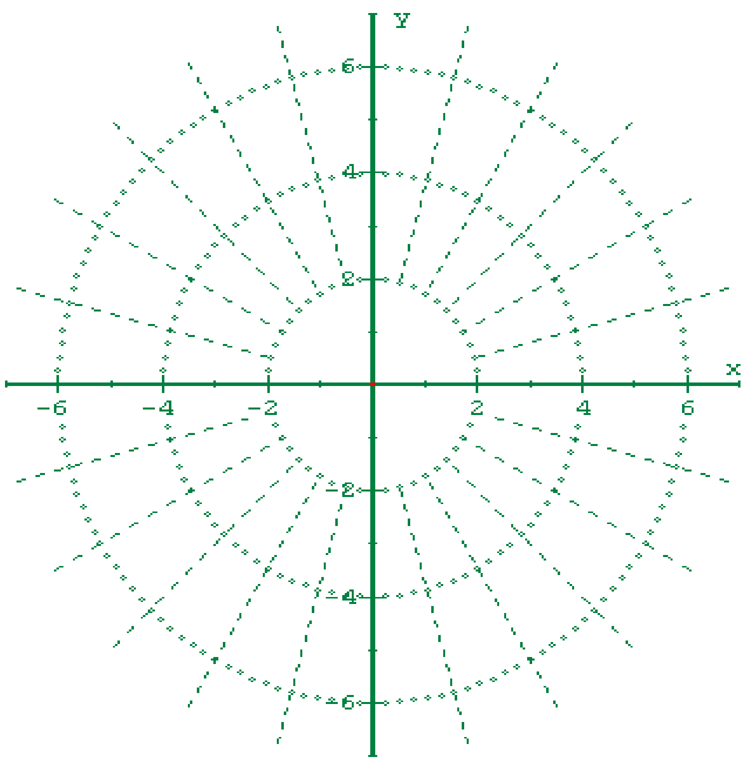
1. Find the derivative of $r = 2\sin\theta$

Remember that horizontal tangent lines occur when

and

that vertical tangent lines occur when

2. Find the HTL and VTL for $r = 2(1 - \cos\theta)$ and sketch the graph.



Recall our formula for the derivative of a function in polar coordinates. Since $x = r\cos\theta = f(\theta)\cos\theta$ and $y = r\sin\theta = f(\theta)\sin\theta$, we can conclude that $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} =$

Solutions obtained by setting $r = 0$ gives equations of tangent lines through the pole.

3. Find the equations of the tangent line(s) through the pole if $r = 2\sin 2\theta$.

To find the arc length of a polar curve, you have two options.

1) You can use the parametrization of the polar curve:

$x = r\cos\theta = f(\theta)\cos\theta$ and $y = r\sin\theta = f(\theta)\sin\theta$

then use the arc length formula for parametric curves:

$$\int_{\alpha}^{\beta} \sqrt{(x'(\theta))^2 + (y'(\theta))^2}\, d\theta$$

or,

2) You can use an alternative formula for arc length in polar form:

$$\int_{\alpha}^{\beta} \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta$$

(Sometimes this gives a cleaner integral to solve.)

4. Find the arc length of the curve $r = 4cos^2(\frac{\theta}{2})$

To find the points of intersection of two polar curves $r = f(\theta)$ and $r = g(\theta)$ we will solve a system of equations. Because there is not a unique representation for each polar coordinate we sometimes need to look for alternate forms of each polar equation. An alternate form for a polar equation $r = f(\theta)$ is given by the equation $(-1)^n r = f(\theta + n\pi)$. For example an alternate form for the equation $r = 2 - \sin\theta$ is $-r = 2 - \sin(\theta + \pi)$ which is equivalent to $r = -2 - \sin\theta$.

5. Find the points of intersection of $r = 2\sin 2\theta$ and $r = 1$.

The development of the formula for the area of a polar region is similar to the development of the area of a rectangular region except rather than summing the areas of rectangles we sum the areas of sectors of a circle.

Theorem: If f is continuous and nonnegative on the interval $[\alpha, \beta]$, then the area of the region bounded by the graph of $r = f(\theta)$ and the radial lines $\theta = \alpha$ and $\theta = \beta$ is given by $\frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$

Find the following areas

6. The area enclosed by $r = 2 - \sin\theta$.

7. The area enclosed by $r = 2\cos 3\theta$

8. The area enclosed by one loop of $r^2 = 2\sin 2\theta$

9. The area of the inner loop of $r = 1 - 2\cos\theta$

10. The area outside $r = 2$ and inside $r = 4\cos\theta$

11. The area common to $r = 2 + 2\sin\theta$ and $r = 1$

12. Find the area common to $r = 4\sin\theta$ and $r = 4\cos\theta$.