Polar Coordinates

So far when we have plotted points on a coordinate system to create a graph we have used the rectangular or Cartesian coordinate system. The equations whose solutions we have plotted have been in rectangular or parametric form. In this section we introduce a different way to locate points on a plane. Rather than using a pair of intersecting perpendicular lines (which we called the x & y-axes) as reference, we use a fixed point (*pole)* and a fixed ray (*polar axis*) and refer to points in relationship to these.

In the Cartesian coordinate system the coordinates (x, y) represent the distance along the x-axis and the distance along the y-axis respectively. In the Polar coordinate system the coordinates (r, θ) represent the directed distance from the pole to the point and the directed angle, counterclockwise from the polar axis to the segment from the pole to the point.

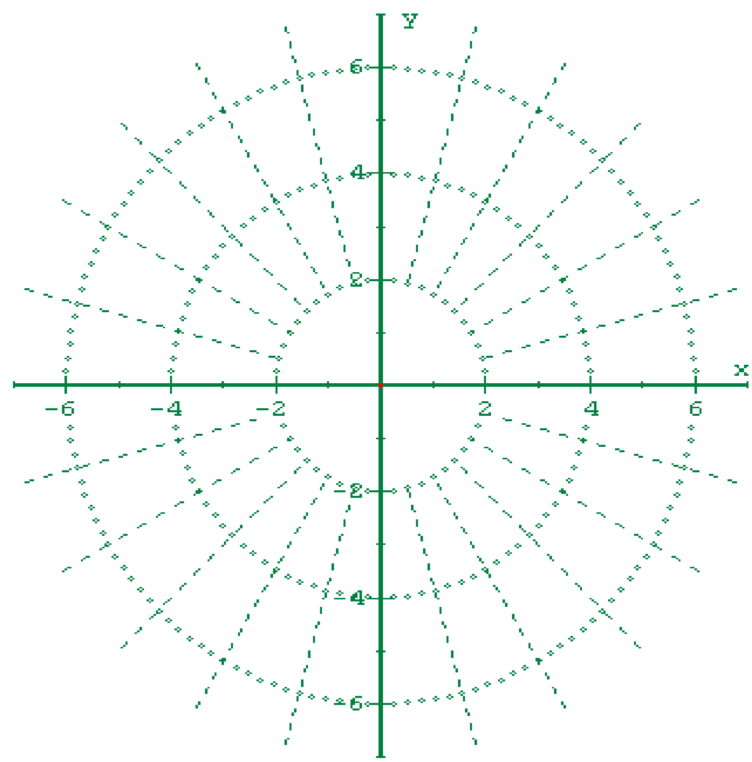
1. Locate each of the following points on the polar coordinate system.

a) $\left(4, \dfrac{\pi}{3}\right)$

b) $\left(4, -\dfrac{2\pi}{3}\right)$

c) $\left(-4, -\dfrac{5\pi}{4}\right)$

d) $\left(1, \dfrac{7\pi}{6}\right)$



With rectangular coordinates, each point (x, y) has a unique representation. This is not true with polar coordinates. Plot the points $\left(2, \frac{\pi}{6}\right)$ and $\left(2, \frac{13\pi}{6}\right)$. Notice that they are the same point.

2. Given the point $\left(3, \frac{5\pi}{6}\right)$, Relabel the point where

a) $r < 0, \theta \geq 0$

b) $r > 0, -2\pi < \theta \leq 0$

c) $r < 0, -2\pi < \theta \leq 0$

3. Given the point $\left(\sqrt{2}, -\frac{\pi}{4}\right)$, Relabel the point with the same r, then relabel with opposite r.

To convert coordinates between the rectangular system and the polar system we will superimpose the systems on top of each other and observe relationships.

Note the following: $x^2 + y^2 =$ $\tan\theta =$

$x =$ $y =$

4. Convert the following to <u>Cartesian coordinates</u>:

a) $\left(-2, -\frac{\pi}{2}\right)$

b) $\left(-1, \frac{\pi}{4}\right)$

c) $\left(2, -\frac{7\pi}{6}\right)$

5. Convert to polar coordinates: ($r > 0, 0 \leq \theta < 2\pi$)

a) $(3, -3)$

b) $\left(-1, \sqrt{3}\right)$

6. Write a polar equation for each of the following. Express in terms of r & θ, not x & y. Express as $r = f(\theta)$

a) $y = 2$

b) $x^2 = 8y$

c) $y = 6x$

7. Write a Cartesian equation for each of the following:

a) $r = 3$

b) $\theta = \frac{\pi}{4}$

c) $r \sin\theta = -2$

d) $r^2 \sin 2\theta = 4$

e) $r = -2\sin\theta$

Following are the graphs of some special polar functions.

Limacons
$r = a \pm bcos\theta$
$r = a \pm bsin\theta$
$(a > 0, b > 0)$

$a + bcos\theta$ $a + bsin\theta$ $a + acos\theta$ $a + asin\theta$

$a - bcos\theta$ $a - bsin\theta$ $a - acos\theta$ $a - asin\theta$

$a + bcos\theta$ $a + bsin\theta$ $a + bcos\theta$ $a + bsin\theta$

$a - bcos\theta$ $a - bsin\theta$ $a - bcos\theta$ $a - bsin\theta$

Rose curves
n petals if n is odd
$2n$ petals if n is even
($n \geq 2$)

$r = acosn\theta$ $r = acosn\theta$ $r = asinn\theta$ $r = asinn\theta$

Circles and Lemniscates

$r = acos\theta$ $r = asin\theta$ $r^2 = a^2sin2\theta$ $r^2 = a^2cos2\theta$

To sketch the graph of a polar function consider the following strategy.

1. Recognize basic forms:

limacon or cardioid $\begin{cases} r = a \pm b\cos\theta \\ r = a \pm b\sin\theta \end{cases}$

rose $\begin{cases} r = a\cos n\theta \\ r = a\sin n\theta \end{cases}$

circle $\begin{cases} r = a\cos\theta \\ r = a\sin\theta \end{cases}$

lemniscate $\begin{cases} r^2 = a^2\cos 2\theta \\ r^2 = a^2\sin 2\theta \end{cases}$

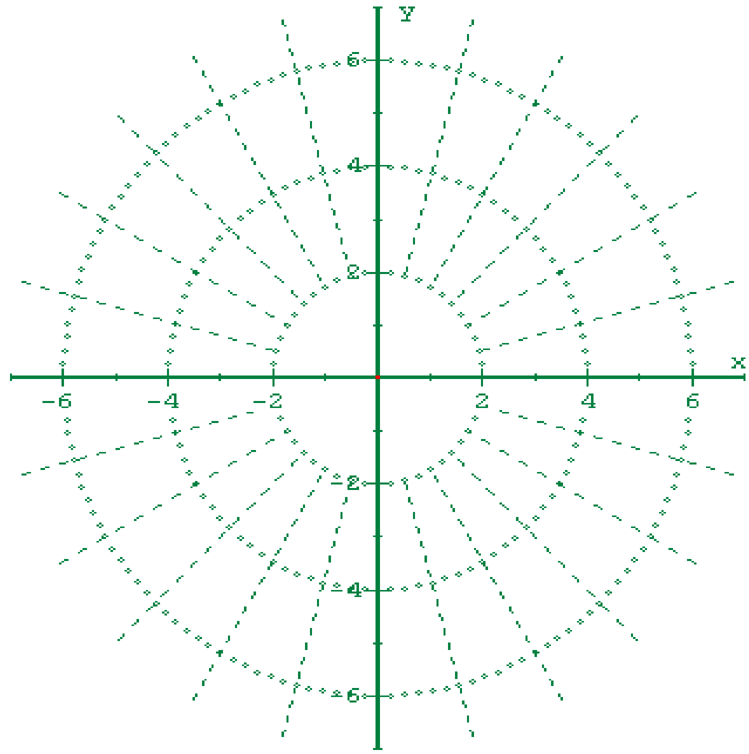
2. Consider symmetry

3. Plot axis points, others if necessary

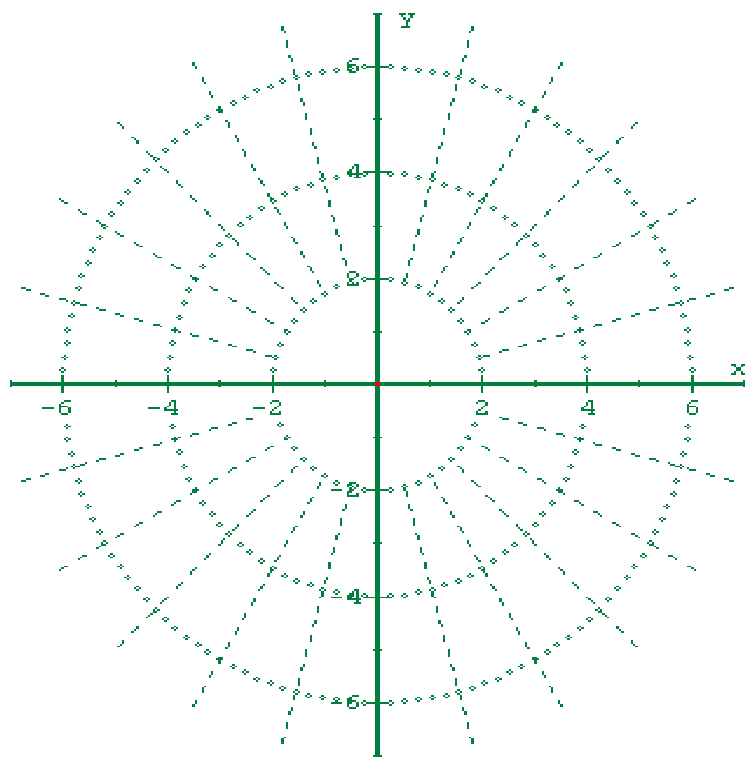
4. Consider the Cartesian form of the equation.

Sketch the graph of each of the following polar equations.

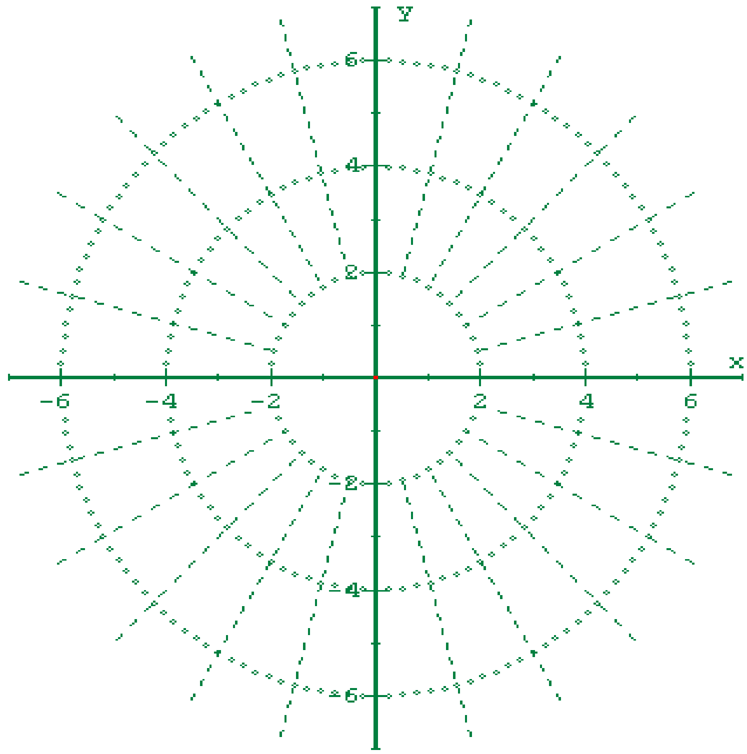
12. $r = 3 - 3\cos\theta$



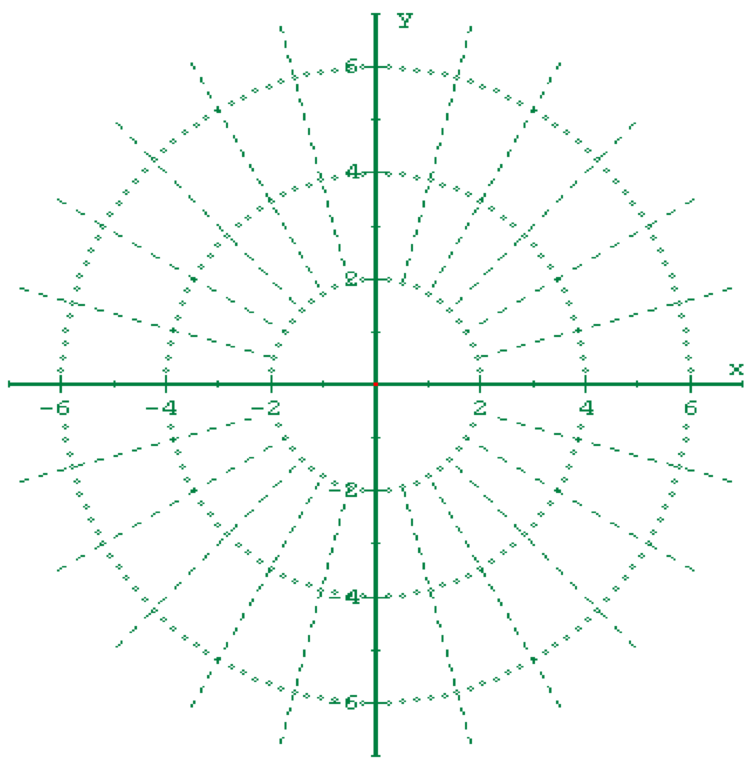
13. $r = 1 + 2\cos\theta$



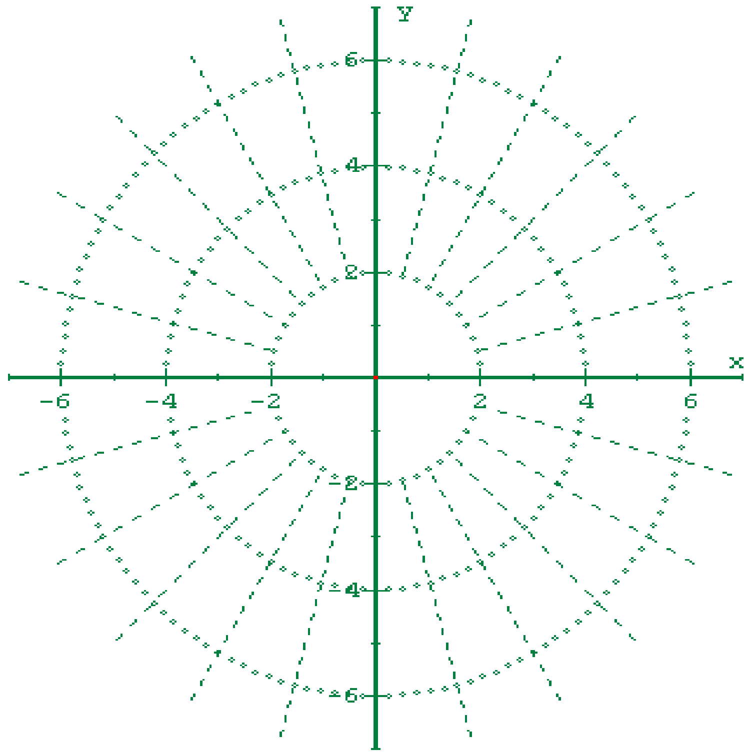
14. $r = 4 + 3\sin\theta$



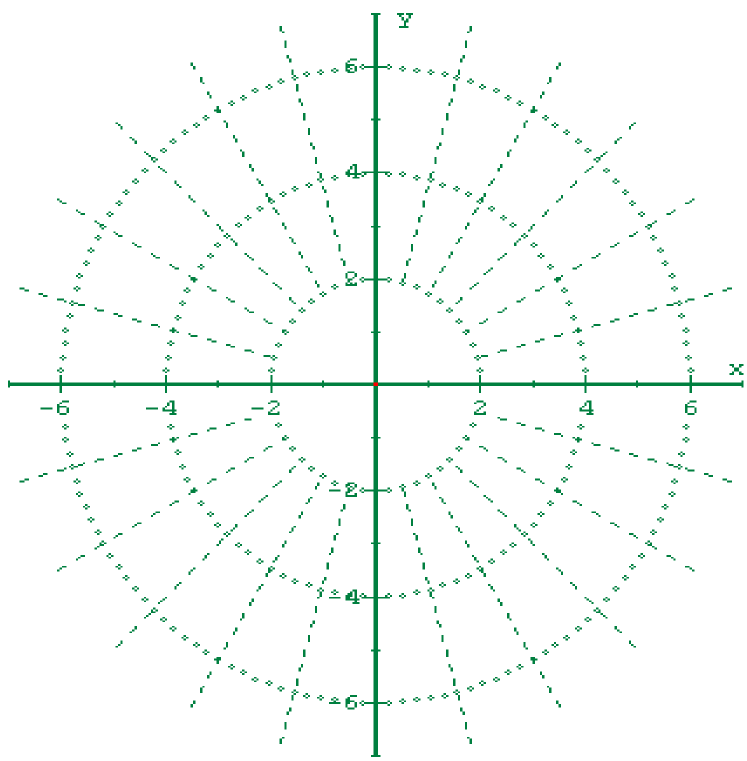
15. $r = 3\cos 2\theta$



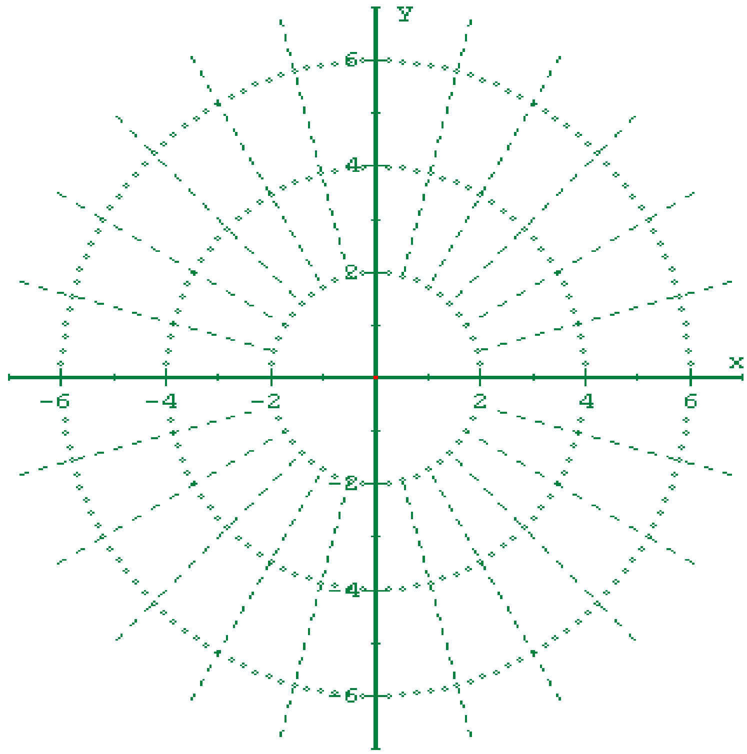
16. $r = 2\sin\theta$



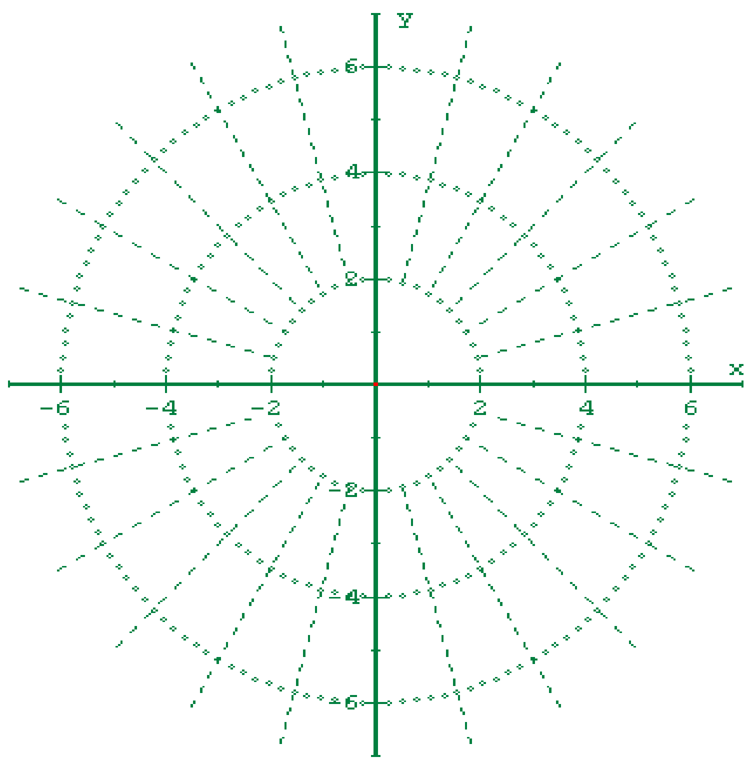
17. $r = 2\cos\theta$



18. $r^2 = 4\sin 2\theta$



19. $r = 2$



20. $\theta = \dfrac{\pi}{3}$