

Powers of Trig functions

In this section we learn how to integrate various powers and combinations of the trigonometric function. Always check for an existing $u du$ relationship first. If there is no apparent $u du$ relationship, think about how you might create one; we often choose part of the integrand as du first, then see if the appropriate u is also present. It is often helpful to use the unit circle identities:

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Examples:

1. $\int \sin^3 x dx$

For odd powers of sine or cosine, save one and convert the others using Pythagorean Identities

2. $\int \cos^2 2x dx$

For even powers of sine or cosine, use half-angle identities

3. $\int \sin^3 2x \cos^3 2x dx$

4. $\int \cos^5 4x dx$

5. $\int \tan^5 x \sec x \, dx$

If tan is odd, save a $\sec x \tan x$
and convert others to $\sec x$

6. $\int \tan^6 2x \sec^4 2x \, dx$

If sec is even, save $\sec^2 x$,
and convert others to $\tan x$

7. $\int \sin^2 x \, dx$

For even sines or cosines,
use half-angle identities

8. $\int \tan^3 x \, dx$

If no secants, convert $\tan^2 x$
to $\sec^2 x$

9. $\int \sin 3x \cos 4x \, dx$

If different angles, use the product to sum identities

$$\sin u \cos v = \frac{1}{2}(\sin(u - v) + \sin(u + v))$$

10. $\int \sin^2 x \cos^2 x \, dx$