Powers of Trig functions

In this section we learn how to integrate various powers and combinations of the trigonometric function. Always check for an existing $u\,du$ relationship first. If there is no apparent $u\,du$ relationship, think about how you might create one; we often choose part of the integrand as du first, then see if the appropriate u is also present. It is often helpful to use the unit circle identities:

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\tan^2 x + 1 = \sec^2 x$ $\qquad$ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\cot^2 x + 1 = \csc^2 x$

Examples:

1. $\displaystyle\int \sin^3 x\,dx$

For odd powers of sine or cosine, save one and convert the others using Pythagorean Identities

2. $\displaystyle\int \cos^2 2x\,dx$

For even powers of sine or cosine, use half-angle identities

3. $\int \sin^3 2x \cos^3 2x \, dx$

4. $\int \cos^5 4x \, dx$

5. $\int \tan^5 x \sec x \, dx$

If tan is odd, save a sec$x$tan$x$
and convert others to sec$x$

6. $\int \tan^6 2x \sec^4 2x \, dx$

If sec is even, save sec$^2 x$,
and convert others to tan$x$

7. $\int \sin^2 x \, dx$

For even sines or cosines, use half-angle identities

8. $\int \tan^3 x \, dx$

If no secants, convert $\tan^2 x$ to $\sec^2 x$

9. $\int \sin 3x \cos 4x \, dx$

If different angles, use the product to sum identities

$\sin u \cos v = \frac{1}{2}(\sin(u-v) + \sin(u+v))$

10. $\int \sin^2 x \cos^2 x \, dx$