Representation of Functions by Power Series

We will begin to develop power series for non-algebraic functions. There are several strategies for developing these power series and we will look at a variety of these strategies and then generalize the process in the next lecture.

1. Consider the series $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$

What kind of a series is this?

For what values of x does the series converge?

What is the sum of the series?

2. Find a power series representation for $\dfrac{3}{1-x}$

One method used to create a new power series from an old power series is to re-center a known power series. Examples illustrating this technique follow.

3. Find a power series representation for $\dfrac{3}{1-x}$ centered at 4.

4. Find a power series for $\dfrac{4}{x+3}$ centered at 0.

5. Find a power series for $\dfrac{4}{x+3}$ centered at 5.

We will derive as many power series as possible by re-centering, substitution, and differentiation or integration of known series. Domains will follow with endpoint considerations.

6. Use substitution and the power series representation for $\dfrac{1}{1-x}$ to find a series for $\dfrac{1}{1+x}$.

7. Use integration to find a power series representation for $\ln(1+x)$

Usefulness:

8. Use the power series representation for $\frac{1}{1-x}$ to create a power series representation for $\ln(1-x)$ centered at 0. Find the domain of the series.

9. Find a power series for $\ln\left(\dfrac{1+x}{1-x}\right)$

10. Use a 7th degree polynomial to approximate $\ln 2$ and determine the truncation error.

11. Create a power series for $\tan^{-1}x$ by using substitution and integration of the power series for $\dfrac{1}{1+x}$.

12. Use the power series created above to approximate $\tan^{-1}\frac{1}{2}$ with error less than $.001$.

13. Evaluate $\int_0^{\frac{3}{4}} \tan^{-1} x^2 \, dx$ with $|R_n| < .001$

14. How many terms will you need to approximate the value of π with an error less than $.0001$ if you use $\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$?

Note: $\frac{\pi}{4} = \tan^{-1} 1$