Sequences

Sequence: A function whose domain is the set of positive integers.

$f(n) = \frac{1}{n}$ or $a_n = \frac{1}{n}$

Examples: Write the first 6 terms of each of the following sequences.

1. $a_n = \sin \dfrac{n\pi}{2}$

2. $a_n = \dfrac{3n!}{(n-1)!}$

3. $a_1 = 4,\ a_{n+1} = \left(\dfrac{n+1}{2}\right)a_n$

4. Write the next two apparent terms in the sequence $\dfrac{7}{2}, 4, \dfrac{9}{2}, 5, \ldots$

5. Simplify: $\dfrac{25!}{23!}$ $\dfrac{(n+2)!}{n!}$ $\dfrac{(2n+2)!}{(2n)!}$

Write the n^{th} term for each of the following sequences:

6. $3, 7, 11, 15. \ldots$

7. $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \ldots$

8. $1+\frac{1}{2}, 1+\frac{3}{4}, 1+\frac{7}{8}, 1+\frac{15}{16}, \ldots$

9. $1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \ldots$

Corollary: If $\lim_{x\to\infty} f(x) = \infty$, then the sequence a_n diverges. If $\lim_{x\to\infty} f(x) =$ DNE for any other reason, then no conclusion can be drawn concerning the sequence without further investigation.

Recall: $\lim_{n\to\infty} x^n = 0$ if $|x| < 1$; $\lim_{n\to\infty} x^n =$ DNE if $x \leq -1$

$\lim_{n\to\infty} x^n = 1$ if $x = 1$; $\lim_{n\to\infty} x^n = \infty$ if $x > 1$

Write the first 5 terms of each of the following (or draw the graph of the associated function to visualize), then determine the convergence or divergence of each. If convergent, find the limit.

10. $a_n = \dfrac{1}{n^{\frac{3}{2}}}$

11. $a_n = \dfrac{\sqrt{n}}{\sqrt{n}+1}$

12. $a_n = \dfrac{\ln n^2}{n}$

13. $a_n = \dfrac{(n-2)!}{n!}$

14. $a_n = 1 + (-1)^n$

15. $a_n = \{\sin 2\pi n\}$

16. $a_n = \cos 2\pi n$

17. $a_n = \cos \pi n$

Use the squeeze theorem, if possible, to determine convergence/divergence of the sequence.

18. $a_n = \dfrac{\cos n}{n}$

19. $a_n = n \sin \frac{1}{n}$

A sequence is **monotonic** if the terms are nondecreasing or nonincreasing.

Examples: Determine whether the following sequences are increasing, decreasing, or neither.

20. $a_n = 3 + (-1)^n$

21. $a_n = \frac{\ln n}{n}$

22. $a_n = \frac{2n}{n+1}$

23. $a_n = = \left\{ \frac{1 \cdot 3 \cdot 5 \cdot \ \ldots (2n-1)}{2^n n!} \right\}$

24. $a_n = \dfrac{n!}{2^n}$

Factorials grow faster than exponential functions.

Theorem: If a sequence $\{a_n\}$ is bounded and monotonic, then it converges.

Examples: Show that each of the following sequences converge.

25. $\left\{ \frac{3^n}{n!} \right\}$

Factorials grow faster than exponentials.

26. $\left\{ \dfrac{n^2}{2^n} \right\}$

Exponentials grow faster than polynomials.