

## Trigonometric Substitution

When you have quadratic expressions in the integrand and no other method of integration works making a trigonometric substitution will sometimes turn the integral into a form that we can handle.

Below are two examples that have quadratic forms in the integrand that we can integrate and one that we cannot currently integrate.

1.  $\int \frac{x \, dx}{x^2 + 2x + 3}$

2.  $\int \frac{dx}{\sqrt{2x - x^2}}$

3.  $\int \sqrt{9 - x^2} \, dx$

When the integrand has an expression including  $\sqrt{a^2 - u^2}$   $a > 0$ , making the substitution  $u = a \sin \theta$  will often times enable us to transform the integrand into an expression that we can integrate.

$$\sqrt{a^2 - u^2} =$$

4. 
$$\int \frac{\sqrt{4 - x^2}}{x^2} dx$$

When the integrand has an expression including  $\sqrt{u^2 - a^2}$   $a > 0$ , making the substitution  $u = a \sec \theta$  will often times enable us to transform the integrand into an expression that we can integrate.

$$\sqrt{u^2 - a^2} =$$

5. 
$$\int \frac{dx}{x^3 \sqrt{x^2 - 25}}$$

When the integrand has an expression including  $\sqrt{u^2 + a^2}$   $a > 0$ , making the substitution  $u = a \tan \theta$  will often times enable us to transform the integrand into an expression that we can integrate.

$$\sqrt{u^2 + a^2} =$$

6. 
$$\int \frac{dx}{\sqrt{x^2 + 9}}$$

7.  $\int \frac{x^2 dx}{(x^2 + 1)^2}$

**8.**  $\int \frac{x^2 dx}{(1 - 9x^2)^{\frac{3}{2}}}$

9.  $\int \frac{dx}{(x^2 - 4x)^{\frac{3}{2}}}$

10.  $\int \frac{\sqrt{4x^2 + 9}}{x^4} dx$

**11.**  $\int_{\sqrt{3}}^2 \frac{\sqrt{x^2 - 3}}{x} dx$