Trigonometric Substitution

When you have quadratic expressions in the integrand and no other method of integration works making a trigonometric substitution will sometimes turn the integral into a form that we can handle.

Below are two examples that have quadratic forms in the integrand that we can integrate and one that we cannot currently integrate.

1. $\displaystyle\int \frac{x\ dx}{x^2+2x+3}$

2. $\displaystyle\int \frac{dx}{\sqrt{2x-x^2}}$

3. $\displaystyle\int \sqrt{9-x^2\,dx}$

When the integrand has an expression including $\sqrt{a^2-u^2}$ $a>0$, making the substitution $u=a\sin\theta$ will often times enable us to transform the integrand into an expression that we can integrate.
$\sqrt{a^2-u^2}=$

4. $\displaystyle\int \frac{\sqrt{4-x^2}}{x^2}\,dx$

When the integrand has an expression including $\sqrt{u^2 - a^2}$ $a > 0$, making the substitution $u = a\sec\theta$ will often times enable us to transform the integrand into an expression that we can integrate.
$\sqrt{u^2 - a^2} =$

5. $\displaystyle\int \frac{dx}{x^3\sqrt{x^2 - 25}}$

When the integrand has an expression including $\sqrt{u^2+a^2}$ $a>0$, making the substitution $u = a\tan\theta$ will often times enable us to transform the integrand into an expression that we can integrate.

$\sqrt{u^2+a^2} =$

6. $\displaystyle\int \frac{dx}{\sqrt{x^2+9}}$

7. $\displaystyle\int \frac{x^2 dx}{(x^2+1)^2}$

8. $\displaystyle\int \frac{x^2\,dx}{(1-9x^2)^{\frac{3}{2}}}$

9. $\displaystyle\int \frac{dx}{(x^2-4x)^{\frac{3}{2}}}$

10. $\displaystyle\int \frac{\sqrt{4x^2+9}}{x^4}\,dx$

11. $\displaystyle\int_{\sqrt{3}}^{2} \frac{\sqrt{x^2-3}}{x}\,dx$