Work

Work - the measure of energy needed to perform certain jobs: a truck pulling a load, a crane lifting an object, compressing or stretching a spring, propelling a rocket, . . .

Force: A push or a pull.

It changes the state of rest or state of motion of a body.

Work: (assuming constant force)

$$W = F \cdot D$$

The amount of work required to move a 100 pound object a distance of 3 feet:

Units:

U.S.

Work: foot-pounds (ft-lb)
inch-pounds
foot-tons

Metric (C-G-S):

Force: **Dyne** - the force required to produce an acceleration of 1 cm/sec^2 on a mass of 1 gram

Work: dyne-centimeters (ergs)
newton-meters (joules) 1 joule = 10^7 ergs

Variable force: If the amount of force changes as the object changes position, this is variable force. Calculus is needed!

Hooke's Law: (1635-1703)
The force F required to stretch or compress a spring (within its elastic limits) is proportional to the distance x that the spring is stretched or compressed from its natural length. $\boldsymbol{F = k \cdot x}$

Examples:

1. A force of 5 pounds compresses a 15 inch spring a total of 4 inches. How much work is done in compressing the spring from a length of 10 inches to a length of 6 inches?

2. How much work is done to stretch a 15 inch spring from a length of 20 inches to a length of 24 inches if a force of 5 pounds stretches the spring 4 inches?

Work done to move fluids:

ΔW = (Force increment) (Distance) = (ΔF) (distance)

Typically in these problems, you want to think of

Force = the weight of slice

You may need to use the fact that

Weight = Density x Volume

3. A fish tank with a rectangular base having width 2 feet, length 4 feet and height 3 feet is full of water. Find the work done to pump the water over the top edge to empty the tank. (Density of water is 62.4 LB/ft ^{3})

4. A right circular conical tank (vertex down) has radius of 5 feet and a height of 20 feet. If the tank is full of water find the work done to pump the water out over the top of the tank.

5. Using the tank in example 4, find the amount of work done to fill the tank if the water source is at the base of the tank.

6. Using the tank in example 4, find the work done to fill the tank if the source is 15 feet below the tank.

7. The top of a cylindrical storage tank for gasoline at a service station is 4 feet below ground level. The axis of the tank is horizontal and its diameter and length are 10 feet and 12 feet, respectively. Find the work done in pumping the entire contents of the full tank to a height 3 feet above ground level. (gas weighs 42 LB/ft^3)

(Don't measure y from bottom up, better to center circle at the origin and measure y from top down or center up.)

Work to move chains, cables, etc.

ΔW=(Force)(Distance increment)=(F)(Δx or Δy)

8. Find the work done to wind a chain 100 feet long, weighing 10 pounds.

9. When is one half of the work done?

10. Find the work done to wind the chain $\frac{3}{4}$ of the way.

11. Find the work done to raise the chain 40 ft. above ground assuming the chain has a 500 pound weight attached.

12. A 100 foot long chain weighs 5 pounds and has a 20 pound bucket containing 60 pounds of sand attached. Find the work done in winding the chain.

13. What if the sand is leaking out at a rate of $\frac{1}{5}$ lb/ft?

14. What if the chain weighs $\frac{1}{2}$ lb/ft and 40 lb. of sand have leaked out when the chain is fully wound?

15. What if the chain weighs 8 lb. and 10 lb. of sand are left when the chain is fully wound?