Geometric Sequences and Series Homework

Find the common ratio for each geometric sequence:

1. $5, 15, 45, 135, \ldots$
2. $-15, 30, -60, 120, \ldots$
3. $3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \ldots$
4. $4, -0.4, 0.04, -0.004, \ldots$

Write the first five terms of each geometric sequence with the given first term, a_1, and common ratio, r.

5. $a_1 = 2$, $r = 3$
6. $a_1 = 20$, $r = \frac{1}{2}$
7. $a_1 = -4$, $r = -10$
8. $a_1 = -\frac{1}{4}$, $r = -2$

Use the formula for the general term (the nth term) of a geometric sequence to find the indicated term of each sequence with given first term, a_1, and common ratio, r.

9. Find a_8 when $a_1 = 6$, $r = 2$.
10. Find a_{12} when $a_1 = 5$, $r = -2$.
11. Find a_6 when $a_1 = 6400$, $r = -\frac{1}{2}$.
12. Find a_8 when $a_1 = 1000000$, $r = 0.1$.

Write a formula for the general term (the nth term) of each geometric sequence. Then use the formula for a_n to find a_7, the seventh term of the sequence.

13. $3, 12, 48, 192, \ldots$
14. $18, 6, 2, \frac{2}{3}, \ldots$
15. $1.5, -3, 6, -12, \ldots$
16. $0.0004, -0.004, 0.04, -0.4, \ldots$

Use the formula for the sum of the first n terms of a geometric sequence to solve the following:

17. Find the sum of the first 12 terms of the geometric sequence: $2, 6, 18, 54, \ldots$
18. Find the sum of the first 11 terms of the geometric sequence: $3, -6, 12, -24, \ldots$
19. Find the sum of the first 14 terms of the geometric sequence: $-\frac{3}{2}, 3, -6, 12, \ldots$

Find the indicated sum:

20. $\displaystyle\sum_{i=1}^{8} 3^i$

21. $\displaystyle\sum_{i=1}^{10} 5 \cdot 2^i$

22. $\displaystyle\sum_{i=1}^{6} \left(\tfrac{1}{2}\right)^{i+1}$

Determine whether the sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference. If the sequence is geomtric, find the common ratio.

23. $a_n = n + 5$

24. $a_n = 2^n$

25. $a_n = n^2 + 5$

26. You are offered a job that pays $30,000 for the first year with an annual increase of 5% per year beginning in the second year. That is, beginning in year 2, your salary will be 1.05 times what it was in the previous year. What can you expect to earn in your sixth year on the job? Round to the nearest dollar.

27. You are investigating two employment opportunities. Company A offers $30,000 the first year. During the next four years, the salary is guaranteed to increase by 6% per year. Company B offers $32,000 the first year, with guaranteed annual increases of 3% per year after that. Which company offers the better total salary for a five-year contract? By how much? Round to the nearest dollar.

Answers:

1. $r = 3$
2. $r = -2$
3. $r = \frac{3}{2}$
4. $r = -0.1$
5. $2, 6, 18, 54, 162$
6. $20, 10, 5, \frac{5}{2}, \frac{5}{4}$
7. $-4, 40, -400, 4000, -40000$
8. $-\frac{1}{4}, \frac{1}{2}, -1, 2, -4$
9. $a_8 = 768$
10. $a_{12} = -10240$
11. $a_6 = -200$
12. $a_8 = 0.1$
13. $a_n = 3(4)^{n-1}; a_7 = 12288$
14. $a_n = 18(\frac{1}{3})^{n-1}; a_7 = \frac{2}{81}$
15. $a_n = 1.5(-2)^{n-1}; a_7 = 96$
16. $a_n = 0.0004(-10)^{n-1}; a_7 = 400$
17. $531, 440$
18. 2049
19. $\frac{16383}{2}$ or 8191.5
20. 9840
21. 10230
22. $\frac{63}{128}$
23. arithmetic, $d = 1$
24. geomtric, $r = 2$
25. neither
26. You can expect to earn approximately \$38,288 in your sixth year on the job.
27. Company B offers the better total salary by approximately \$780.