Infinite Sequences and Series

An infinite sequence is a function whose domain is the set of natural numbers.

Example: $f(n) = \frac{1}{n}$ or $a_n = \frac{1}{n}$ or $\left\{\frac{1}{n}\right\}$

n ranges over the set $\{1, 2, 3, 4, 5, \ldots\}$

a_n = the nth term

Examples: Write the first 3 terms of each of the following sequences whose general terms are given:

1. $a_n = 2n+1$

2. $a_n = (-1)^n \frac{n}{3n+1}$

Factorial Notation: If n is a positive integer, the notation n! is the product of all positive integers from n down through 1.

0! = **1! =** **2! =** **3! =** **4! =** **5! =**

Find the first 3 terms of the sequence:

$$\frac{3^n}{(n+1)!}$$

$$\frac{(-1)^{n+1}}{(2^n+1)!}$$

The population size of a culture of bacteria triples every hour such that its size is modeled by the sequence

$$a_n = 50(3)^{n-1}$$

where n is the number of the hour just beginning. Find the size of the culture at the beginning of the fourth hour and the size of the culture at the beginning of the first hour.

The sum of n terms $a_1, a_2, a_3, a_4, \ldots$ **is written**

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$$

i **is the index of summation,** a_i **is the** i**th term of the sum and the upper and lower limits of summation are 1 and n.**

Examples:

1. $\sum_{i=3}^{6} i^2 =$

2. $\sum_{k=3}^{6} k^2 =$

3. $\sum_{k=1}^{5} 6 =$

4. $\sum_{k=1}^{4} \left(-\frac{1}{3}\right)^k =$

5. $\sum_{i=1}^{5} \frac{(i+2)!}{i!} =$

Write the following sum as an infinite series in summation notation:

1. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots + \frac{9}{10}$

2. $1 - 2 + 4 - 8 + 16 - 32$

3. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \frac{1}{49}$