

Math 940 Final Exam Review

Course Learning Objectives:

- 1) Solve certain types of linear, quadratic, polynomial, rational, radical, exponential and logarithmic equations and inequalities.
- 2) Model applications based on the types of equations and inequalities listed in 1).
- 3) Demonstrate appropriate manipulation of rational expressions, expressions with rational exponents and radicals, and exponential and logarithmic expressions.
- 4) Graph linear, quadratic, exponential and logarithmic equations and model applications based on these equations and their graphs.
- 5) Demonstrate appropriate manipulation of function notation and be able to find the domain, range, inverse and transformations of a function.
- 6) Solve systems of linear equations using matrices, and modeling applications based on linear systems.

1. Express in interval notation: $x > -3$ **OR** $x < 3$.
2. Express in interval notation: $x \geq 1$ **AND** $x < 4$
3. The solution set for the absolute value inequality $|3x - 1| < -2$ is ...
4. Find the center and radius of the circle by rewriting the equation in standard form.
$$x^2 + y^2 + 4x - 2y = 4$$
5. $(-64)^{-2/3} = \dots$
6. If $i^2 = -1$, then $(4 - 3i)^2 = \dots$
7. $\sqrt[9]{y^6} = \dots$
8. Rationalize the denominator: $\frac{3}{\sqrt[3]{4}} = \dots$
9. Simplify: $\frac{\sqrt[3]{a}}{\sqrt[4]{a}} = \dots$
10. Factor: $64x^3 - 27 = \dots$

11. Solve for x : $3^{x+1} = 27^{2x-7}$
12. If $\ln x = -4$, then $x = \dots$
13. If $\log x = -2$, then $x = \dots$
14. $2 \log x - 3 \log y =$
15. If $f(x) = -2x^2 + 3x - 1$, then $f(-2) = \dots$
16. If $f(x) = 2x^2 - x - 2$, then $f(a + h) = \dots$
17. The range of the linear function $f(x) = 1 + 2x$ is \dots
18. The domain of the rational function $f(x) = \frac{x + 1}{x - 3}$ is \dots
19. The range of the quadratic function $f(x) = -2(x + 2)^2 - 1$ is \dots
20. Find the value of $f(1)$ if $f(x) = \begin{cases} 2x - 1 & \text{if } x < -2 \\ -3 & \text{if } -2 \leq x \leq 2 \\ 2x + 1 & \text{if } x > 2 \end{cases}$
21. Solve the following absolute value inequality.
Write your solution in interval notation.

$$|3x - 1| \geq 5$$

22. Simplify the complex fraction completely:

$$\frac{\frac{1}{3} + \frac{1}{x}}{\frac{x}{3} - \frac{3}{x}}$$

23. Perform the indicated operations and simplify:

$$\frac{1}{4-x^2} + \frac{x}{x^2+x-6} - \frac{1}{x+3}$$

24. Find all **four** solutions of the following polynomial equation.

$$x^4 + 21x^2 - 100 = 0$$

25. Set up a 2×2 linear system to solve the following word problem, then solve the problem. Make certain that you carefully define both of your variables and that you clearly answer the question asked.

John has \$10,000 to invest. If the bank offers 2% and 3% simple interest and the total annual interest income from the two accounts is \$225, how much was invested in each account?

26. Solve the following radical equation.

$$\sqrt{z+1} - \sqrt{z-2} = 1$$

27. Use matrices to solve the following 3×3 linear system for (x, y, z) .

$$\left\{ \begin{array}{l} x + 3z = 10 \\ 2x - y - z = -1 \\ -x + 2y + z = 2 \end{array} \right.$$

28. Solve the following rational inequality.
Write your solution in interval notation.

$$\frac{3x - 7}{x - 2} \leq 2$$

29. Solve for x in the following logarithmic equation.

$$\ln 2x - \ln(x - 2) = 0$$

30. A woman can row a boat 3 miles downstream and return a distance of 3 miles upstream in a total time of 2 hours. If the rate of the current is 2 mph, what is the woman's rate of rowing in still water?

31. A cliff measures a height of 1200 feet. A ball is thrown upward from the top of the cliff with an initial velocity of 10 feet per second. The height of the ball h in feet after t seconds is given by the equation:

$$h(t) = -16t^2 + 10t + 1200$$

- a. Find the maximum height of the ball.
- b. How long after the ball is thrown will it hit the ground?
Round to the nearest tenth of a second.
32. Let $f(x) = 3x^2 - 2x + 1$.
Find and simplify completely the difference quotient $\frac{f(a+h)-f(a)}{h}$
33. Given that $f(x) = \sqrt[3]{3x - 1} + 2$ is a one-to-one function, find a formula for the inverse function.

34. The cost of a hospital stay is given by the function:

$$f(x) = 324.5x + 1254$$

where x is the number of days spent in the hospital.

- a. Find the y -intercept of this linear function.
 - b. Interpret the meaning of the y -intercept in this problem situation.
 - c. Find the slope of this linear function.
 - d. Interpret the meaning of the slope in this problem situation.
 - e. Predict the cost of a hospital stay for 2 weeks.
35. The value of a car bought in 2000 depreciates, or decreases, as time passes. Two years after the car was bought, it was worth \$12500; 10 years after it was bought, it was worth \$750.
- a. Write two ordered pairs that can be used to represent this data. Let x represent the number of years **after** 2000.
 - b. Write the **linear function** that models this data.
 - c. Graph the linear function that models this data. Indicate the scale used on your axes.
36. Graph the following exponential function as carefully as possible below. Give exact coordinates of at least three points.

$$f(x) = 3^x - 1$$

Then list the domain and range in interval notation.

37. Graph the following logarithmic function as carefully as possible below. Give exact coordinates of at least three points.

$$f(x) = \log_4 x$$

Then list the domain and range in interval notation.

38. Let $f(x) = \sqrt{x+2} - 3$.
List the transformations being applied and graph $f(x)$.

39. Graph the following quadratic function as carefully as possible below.

$$f(x) = -2x^2 + 8x - 3$$

40. Graph the piecewise defined function

$$f(x) = \begin{cases} 2x - 1 & \text{if } x \leq 1 \\ x + 1 & \text{if } x > 1 \end{cases}$$

Answers:

1. $(-\infty, \infty)$
2. $[1, 4)$
3. \emptyset
4. $C(-2, 1), r = 3$
5. $\frac{1}{16}$
6. $7 - 24i$
7. $\sqrt[3]{y^2}$
8. $\frac{3\sqrt[3]{2}}{2}$
9. $\sqrt[12]{a}$
10. $(4x - 3)(16x^2 + 12x + 9)$
11. $\frac{22}{5}$
12. $\frac{1}{e^4}$
13. $.01$
14. $\frac{x^2}{y^3}$
15. -15
16. $2a^2 + 4ah + 2h^2 - a - h - 2$
17. $(-\infty, \infty)$
18. $(-\infty, 3) \cup (3, \infty)$
19. $(-\infty, -1]$
20. -3
21. $(-\infty, -\frac{4}{3}] \cup [2, \infty)$
22. $\frac{1}{x-3}$

23. $\frac{x+1}{(x^2-4)(x+3)}$
24. $\pm 5i, \pm 2$
25. \$2500 at 3%, \$7500 at 2%
26. 3
27. (1,0,3)
28. (2,3]
29. \emptyset
30. 4
31. a) 1201.56 ft, b) 8.98 sec
32. $6a + 3h - 2$
33. $f^{-1}(x) = \frac{(x-2)^3+1}{3}$
34. a) 1254, b) initial cost of stay is \$1254, c) 324.5 d) \$324.5 per day to stay e) \$5797
35. a) (2, 12,500) (10,750) b) $f(x) = -1468.75x + 15437.5$
 c) graph line - pick good scale
36. D: $(-\infty, \infty)$ R: $(-1, \infty)$ Plot graph through $(-1, -\frac{2}{3}), (0, 0), (1, 2)$
37. D: $(0, \infty)$ R: $(-\infty, \infty)$ Plot graph through (4,1) (1,0) $(\frac{1}{4}, -1)$
38. Left 2, down 3 Plot graph through $(-2, -3) (-1, -2) (2, 1)$
39. V : (2, 5) Plot parabola pointing down - over 1 down 2 on each side of vertex
40. open circle at (1,2) closed circle at (1,1)

