Final Review Problems

Math 940

Course Learning Objectives:

1) Solve certain types of linear, quadratic, polynomial, rational, radical, exponential and logarithmic equations and inequalities.
2) Model applications based on the types of equations and inequalities listed in 1).
3) Demonstrate appropriate manipulation of rational expressions, expressions with rational exponents and radicals, and exponential and logarithmic expressions.
4) Graph linear, quadratic, exponential and logarithmic equations and model applications based on these equations and their graphs.
5) Demonstrate appropriate manipulation of function notation and be able to find the domain, range, inverse and transformations of a function.
6) Solve systems of linear equations using matrices, and modeling applications based on linear systems.

Solve the equation or inequality in problems 1 - 33.

1. $|8-3x|=4$

2. $|2x-7|>14$

3. $|x-7|=|2x+4|$ Also review special cases of 1,2, & 3.

4. $11\le 5x+2<18$

5. $3-4x<-1$ OR $3-4x\ge 9$

6. $x^2-3x<10$

7. $(x+4)^2=32$

8. $2x^2+8x-3=0$ Complete the square.

9. $y^2+20y=-6$ Use the quadratic formula.

10. $(4m+1)(4m-1)=15m^2+8$

11. $6x^2-27=0$

12. $3x^3+7x^2-12x-28=0$

13. $a^4+7a^2-18=0$

14. $(x+4)^2-8(x+4)+15=0$

15. $y^{\frac{2}{3}}-5y^{\frac{1}{3}}-6=0$

16. $m^3+1=0$

17. $\dfrac{x-1}{x-4}>0$

18. $\dfrac{6}{m-1}\ge 1$

19. $\sqrt{2x+9}+2=0$

20. $\sqrt{x+9}=2+\sqrt{x-7}$

21. $\sqrt[3]{x-4}-5=-7$

22. $A=2HW+2LW+2LH$ Solve for H.

23. $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{z}$ Solve for z.

24. $a(1+x)=yx$ Solve for x.

25. $\dfrac{3x}{x-3} - \dfrac{18}{x^2-4x+3} = \dfrac{11}{x-1}$

26. $4 - \dfrac{11}{x} - \dfrac{3}{x^2} = 0$

27. $6^{2x-3} = 216$

28. $\log_4 12 + \log_4 x = 2$

29. $2^{3x-4} = 10$

30. $\log x = -1.15$

31. $\log_x \dfrac{1}{16} = -4$

32. $x = \log_3 27$

33. $e^{3x-5} = 7$ State exact solution and solution rounded to 2 decimal places.

Solve the system of equations in problems 34 & 35.

34.
$$\begin{array}{rcrcl} 2x & - & 5y & = & 12 \\ x & + & 4y & = & 6 \end{array}$$

35.
$$\begin{array}{rcrcrcl} x & + & y & + & z & = & 1 \\ 2x & & & + & z & = & 4 \\ -x & + & 2y & + & 4z & = & 3 \end{array}$$

Perform the indicated operations in problems 36 - 39.

36. $\dfrac{3x}{x+1} + \dfrac{4}{1-x} - \dfrac{6}{x^2-1}$

37. $\dfrac{6x^2-3xy}{2x-1} \cdot \dfrac{4x^2-1}{y-2x}$

38. $\dfrac{5x-15}{3x+6} \div \dfrac{x^2+x-12}{x^2+7x+10}$

Factor the expression in problems 39 - 43.

39. $8x^3 + 1$
40. $(2x + y)^2 + 4(2x + y) + 4$
41. $28r^{-\frac{1}{4}} + 14r^{\frac{1}{2}}$
42. $x^3 + 7x^2 - 4x - 28$
43. $4y^{2n} + 20y^n + 25$

Multiply the expressions in problems 44 - 45.

44. $(4 + 2i)(3 - 5i)$ This is the product of two complex numbers.
45. $\left(3\sqrt{3} - 4\sqrt{2}\right)\left(\sqrt{3} + 2\sqrt{2}\right)$

Completely simplify the expression in problems 46 - 59.

46. $72^{\frac{1}{2}}$
47. $3\sqrt[3]{81^{-1}}$
48. $\dfrac{12\sqrt[4]{96a^6}}{6\sqrt[4]{6a^2}}$
49. $\dfrac{2}{\sqrt{3} + \sqrt{2}}$
50. $\sqrt{50x} - \sqrt{27x} - \sqrt{32x} + \sqrt{3x}$
51. $\sqrt{\dfrac{1}{(3x)^3}}$
52. $\dfrac{3 + 7i}{2 - 5i}$
53. $\left(\dfrac{1}{8}\right)^{-\frac{2}{3}}$
54. $\dfrac{\frac{t}{4} - \frac{1}{t}}{1 + \frac{t+4}{t}}$

55. $\dfrac{12+2\sqrt{90}}{3}$

56. i^{43}

57. $\sqrt{x^2-4xy+4y^2}$

58. $\dfrac{6a^{-1}}{2a^{-2}-3}$

59. $\sqrt{-4}\sqrt{-36}$

Sketch a graph of the equation or inequality in problems 60 - 70

60. $f(x)=-2x^2+1$

61. $f(x)=-3x^2+2x-1$ Find the vertex and intercepts.

62. $f(x)=\dfrac{1}{2}(x+3)^2-1$

63. $y=3^x$

64. $y=\log_2 x$

65. $6x-9y+18=0$

66. $(x+1)^2+(y-3)^2=25$

67. $3x-2y\le 8$

68. $y=|x-2|+3$

69. $y=-\sqrt{x+2}$

70. $f(x)=\left(\dfrac{1}{2}\right)^x-1$

Use the functions $f(x)=3x^2-x$ and $g(x)=\sqrt{10-3x}$ to answer problems 71 - 78.

71. $f(-3)$

72. $g(2)$

73. $\dfrac{f(x+h)-f(x)}{h}$

74. Find the function which is the inverse of $f(x)=\sqrt[3]{2-x}$.

75. If $f(x)=x^2+3$, then $f(a-3)=$

76. If $f(x)=100$, then $f(0)=$

77. Find the domain of $f(x)=\sqrt{x-2}+3$.

78. Find the domain of $f(x)=\dfrac{3}{2x-1}+1$.

79. Express $\log_4 3 + \log_4 7$ as a single logarithm.

80. Express $\log_2 16 - \log_2 3 + \log_2 6$ as a single logarithm

81. State the equation of the line containing $(-2, 1)$ and $(-4, 6)$ in function form.

82. State the equation of the line perpendicular to $x + 2y = 4$ and containing the point $(3, -4)$ in slope intercept form.

83. Determine the distance between $(3, -5)$ and $(-3, 4)$. Also determine the midpoint of the line connecting the points.

84. Use your calculator to estimate $\log_3 200$ to two decimal places.

85. Find the discriminant for $3x^2 + 2x = -1$. What does the discriminant tell us about the solutions to the equation?

Solve the applications in problems 86 - 94.

86. How long will it take \$5000 to earn \$1500 in interest if it is invested at 8% compounded monthly? Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$

87. An 18 foot ladder is leaning against a house. The distance from the bottom of the ladder to the house is 6 feet less than the distance from the top of the ladder to the ground. Find how far the top of the ladder is from the ground. State the exact answer and also a one-decimal-place approximation.

88. While working together, two political campaign workers can stuff envelopes for the candidate's mailing list in 6 hours. Working alone, one worker can finish the same job in 3 hours less time than the other worker. How long does it take each worker to complete the same job if they work alone?

89. The cost C in dollars of manufacturing x compact discs is given by $C(x) = \frac{1}{2}x^2 - 600x + 200,000$.

a) Find the number of compact discs that must be manufactured to minimize the cost.

b) Find the minimum cost.

90. At an amusement park, the daily sales of ice cream are directly proportional to the square of the daily high temperature. If 3698 ice cream cones are sold on a day when the high temperature is $86°$ Fahrenheit, how many ice creams cones will be sold when it is $75°$ Fahrenheit?

91. Eight flats of impatiens and two flats of marigolds cost \$15.50. Three flats of impatiens and five flats of marigolds cost \$10.02. Find the price per flat of each kind of flower.

92. A jet made a test run of 825 miles in 1.5 hours aided by a 50 mph tailwind. The same plane made a run of 675 miles in the same time against the 50 mph wind. What was the speed of the plane in still air?

93. The Pool Fun Company has learned that by pricing a newly released Fun Noodle at \$3, sales will reach 10,000 Fun Noodles per day during the summer. Raising the price to \$5 will cause the sales to fall to 8,000 Fun Noodles per day. Assuming that the relationship between sales price and number of Fun Noodles is linear,

a. Write two ordered pairs given in this problem situation. Let x represent price and let y represent sales.
b. Find the slope for the linear function.
c. Interpret the slope.
d. Write an equation in **function form** describing the relationship between price and sales.
e. Find the y-intercept of your linear function.
f. Interpret the meaning of the y-intercept in this problem situation.
g. Find the x-intercept of your linear function.
h. Interpret the meaning of the x-intercept in this problem situation.
i. Graph the function for the problem situation on the previous page.
j. Write the implied domain and range of this function.

94. The model $A = 1.6e^{.039t}$ gives the minimum wage, A, in year t, where $t = 0$ represents the year 1970.

a. Use the model to find the minimum wage in the year 1990.
b. Use the model to find the year in which the minimum wage will reach \$$7.50$ per hour.

Answers

1. $\left\{\frac{4}{3},\, 4\right\}$

2. $\left\{\left(-\infty, -\frac{7}{2}\right) \cup \left(\frac{21}{2}, \infty\right)\right\}$

3. $\{-11,\, 1\}$

4. $\left[\frac{9}{5}, \frac{16}{5}\right)$

5. $\left\{(-\infty,\, -\frac{3}{2}] \cup (1, \infty)\right\}$

6. $\{(-2,\, 5)\}$

7. $\left\{-4 \pm 4\sqrt{2}\right\}$

8. $\left\{\frac{-4 \pm \sqrt{22}}{2}\right\}$

9. $\left\{-10 \pm \sqrt{94}\right\}$

10. $\{-3,\, 3\}$

11. $\left\{\pm \frac{3\sqrt{2}}{2}\right\}$

12. $\left\{-\frac{7}{3},\ \pm 2\right\}$

13. $\left\{\pm \sqrt{2},\, \pm 3i\right\}$

14. $\{\pm 1\}$

15. $\{-1,\, 216\}$

16. $\left\{-1,\ \frac{1 \pm \sqrt{3}i}{2}\right\}$

17. $\{(-\infty,\, 1) \cup (4,\, \infty)\}$

18. $\{(1, 7\,]\}$

19. $\emptyset$

20. $\{16\}$

21. $\{-4\}$

22. $\left\{H = \frac{A - 2LW}{2W + 2L}\right\}$

23. $\left\{z = \frac{xy}{y + x}\right\}$

24. $\left\{x = \frac{a}{y - a}\right\}$

25. $\left\{\frac{5}{3}\right\}$

26. $\left\{-\frac{1}{4},\, 3\right\}$

27. $\{3\}$

28. $\left\{\frac{4}{3}\right\}$

29. $\{2.44\}$

30. $\{.07\}$

31. $\{2\}$

32. $\{3\}$

33. $\left\{\frac{5 + \ln 7}{3}\right\},\ \{2.32\}$

34. $\{(6,\, 0)\}$

35. $\{(1,\, -2,\, 2)\}$

36. $\frac{3x - 10}{x - 1}$

37. $-3x(2x + 1)$

38. $\frac{5(x + 5)}{3(x + 4)}$

39. $(2x + 1)(4x^2 - 2x + 1)$

40. $(2x + y + 2)^2$

41. $14r^{-\frac{1}{4}}\left(2 + r^{\frac{3}{4}}\right)$

42. $(x + 2)(x - 2)(x + 7)$

43. $(2y^n + 5)^2$

44. $22 - 14i$

45. $2\sqrt{6} - 7$

46. $6\sqrt{2}$

47. $\dfrac{\sqrt[3]{9}}{3}$

48. $4a$

49. $2\left(\sqrt{3}-\sqrt{2}\right)$

50. $\sqrt{2x}-2\sqrt{3x}$

51. $\dfrac{\sqrt{3x}}{9x^2}$

52. $-1+i$

53. 4

54. $\dfrac{t-2}{8}$

55. $4+2\sqrt{10}$

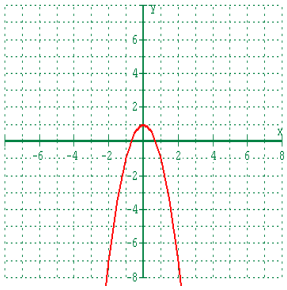
56. $-i$

57. $|x-2y|$

58. $\dfrac{6a}{2-3a^2}$

59. -12

60.



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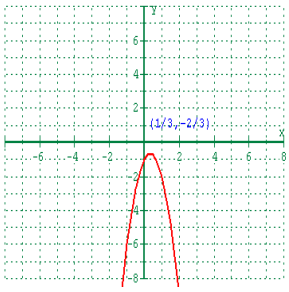

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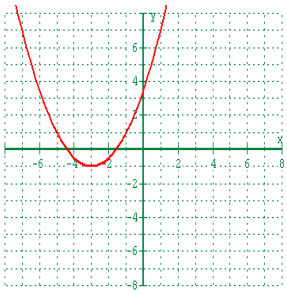

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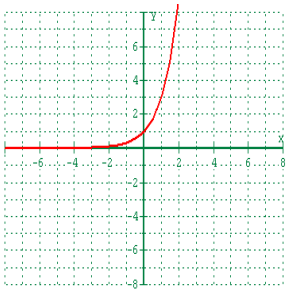

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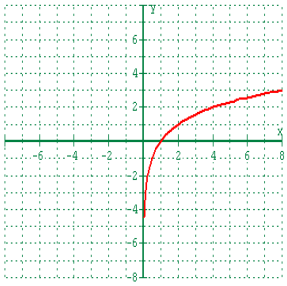

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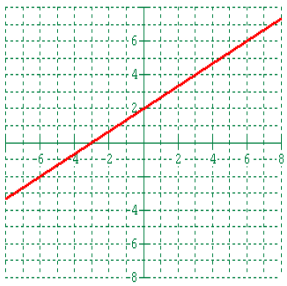

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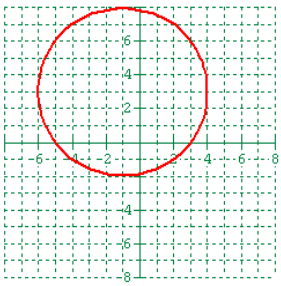

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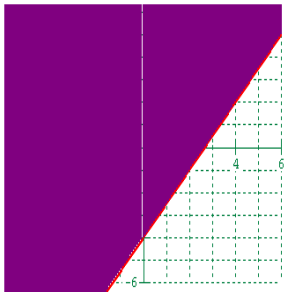

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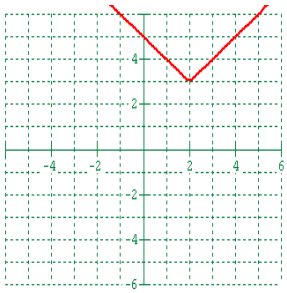

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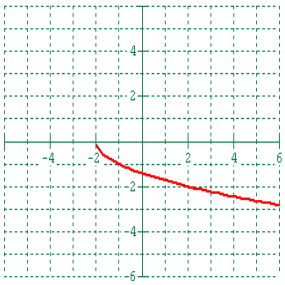

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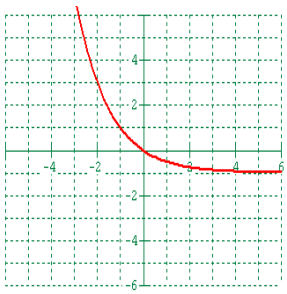


71. 30

72. 2

73. $6x + 3h - 1$

74. $2 - x^3$

75. $a^2 - 6a + 12$

76. 100

77. $[\,2,\,\infty)$

78. $\left(-\infty,\,\frac{1}{2}\right) \cup \left(\frac{1}{2},\,\infty\right)$

79. $\log_4 21$

80. $\log_2 32$

81. $y = -\frac{5}{2}x - 4$

82. $y = 2x - 10$

83. $3\sqrt{13}\,;\ \left(0,\,-\frac{1}{2}\right)$

84. 4.82

85. -8; The negative discriminant indicates that there are two complex solutions.

86. It will take 3.3 years to earn $\$1500$.

87. The top of the ladder is $3 + 3\sqrt{17}$ feet from the ground. This is approximately 15.4 feet.

88. It will take the two workers approximately 13.7 and 10.7 respectively to complete the job if they work separately.

89. a) 600 discs must be manufactured to minimize the cost.
b) The minimum cost is $\$20,000$.

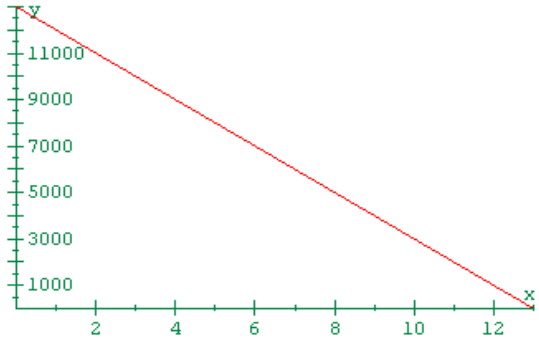
90. 2813 ice cream cones will be sold when the temperature is $75°$.

91. The price per flat is $\$1.69$ for the impatiens and $\$ .99$ for the marigolds.

92. The jet is traveling 500 miles per hour in still air.

93. a. $(3, 10000); (5, 8000)$ b. -1000 c. Sales decrease by 1000 Fun Noodles for every $\$1$ increase in price d. $f(x) = -1000x + 13,000$ e. $(0, 13000)$ f. If you give away the Fun Noodles, you will give away 13000 of them. g. $(13, 0)$ h. If the price of Fun Noodles is $\$13$, you won't sell any.

i.



j. Domain:$[0, 13]$; Range: $[0, 13000]$

94. a. $3.49 b. 2010