

# Evaluating Logs and Properties of Logs Video Lecture

Sections 12.4, 12.5 and 12.6

## Course Learning Objective:

- 1) Solve certain types of logarithmic equations.
- 2) Demonstrate appropriate manipulation of exponential and logarithmic expressions.

## Weekly Learning Objectives:

- 1) Write exponential equations with logarithmic notation and logarithmic equations with exponential notation.
- 2) Solve logarithmic equations by using exponential notation.
- 3) Use the product property of logarithms.
- 4) Use the quotient property of logarithms.
- 5) Use the power property of logarithms.
- 6) Use the properties of logarithms together.
- 7) Identify common logarithms and approximate them with a calculator.
- 8) Evaluate common logarithms of powers of 10.
- 9) Identify natural logarithms and approximate them by a calculator.
- 10) Evaluate natural logarithms of powers of  $e$ .
- 11) Use the change of base formula.

# Evaluating Logarithms and Properties of Logarithms

A logarithm is the inverse of an exponential:

$$y = \log_b x \text{ means } x = b^y$$

We can think of a logarithm (log) as being an exponent.

Exponent Statement

Logarithmic Statement

$$3^2 = 9$$

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$$\log_6 1 = 0$$

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$$4^{-2} = \frac{1}{16}$$

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$$\log_8 2 = \frac{1}{3}$$

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$$e^0 = 1$$

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$$\ln e = 1$$

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Evaluate each of the following logarithmic expressions without a calculator:

$$\log_2 16$$

$$\log_{10} .1$$

$$\log_3 27$$

$$\log_2 \frac{1}{8}$$

$$\log_2 1$$

$$\log_3 \frac{1}{27}$$

$$\log_{\frac{3}{4}} \frac{4}{3}$$

$$\log \sqrt{10}$$

$$\log \frac{1}{100}$$

$$\ln e$$

$$\ln e^2$$

$$\ln \sqrt{e}$$

## Properties of Logarithms:

For any positive real number  $b$ ,  $b \neq 1$

$$\textcircled{1} \log_b b = 1$$

$$\textcircled{2} \log_b 1 = 0$$

$$\textcircled{3} \log_b b^x = x$$

$$\textcircled{4} b^{\log_b x} = x$$

$$\textcircled{5} \log_b(xy) = \log_b(x) + \log_b(y) \quad \text{Product Property}$$

$$\textcircled{6} \log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y) \quad \text{Quotient Property}$$

$$\textcircled{7} \log_b(x)^r = r \log_b(x) \quad \text{Power Property}$$

Note:  $\log_b(x+y) \neq \log_b x + \log_b y$

$$\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right) \underline{\underline{\text{NOT}}} \frac{\log_b x}{\log_b y}$$

Write each of the following as a single logarithm:

$$\log_2 9 + \log_2 3$$

$$\log_7 20 - \log_7 4$$

$$\log_6 18 + \log_6 2 - \log_6 9$$

$$3 \ln 2 + 2 \ln 3$$

$$\log_9 (4x) - \log_9 (x-3) - \log_9 (x^2+1)$$

$$\log x + \log y - \log z + \log w - \log p$$

$$5 \ln x - \frac{1}{2} \ln x + 3 \ln x$$

Write each expression as a sum or difference of logarithms:

$$\log_4 \frac{2}{9z}$$

$$\ln \sqrt{\frac{3}{y}}$$

$$\log_3 \frac{(x+5)^2}{x}$$

$$\log_4 \frac{\sqrt[4]{z} \cdot \sqrt[5]{w}}{t^2}$$

If  $\log_b 3 = .5$  and  $\log_b 5 = .7$ , find:

$$\log_b 25$$

$$\log_b \frac{3}{5}$$

## Change of Base Formula:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Find to four decimal places:

$$\log_3 2$$

$$\log_{\frac{1}{3}} 2$$