Angles and Their Measure Video Lecture

Section 6.1

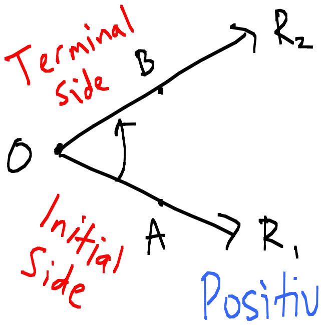
Course Learning Objectives:
Demonstrate an understanding of trigonometric functions and their applications.

Weekly Learning Objectives:

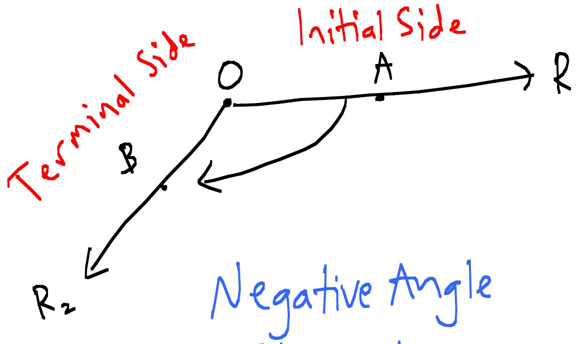
1) Convert from degrees to radians and from radians to degrees.
2) Find coterminal angles.
3) Find the arc length of a circle.
4) Find the area of a sector of a circle.
5) Find the linear and angular speed of an object traveling in circular motion.

Angles and Their Measure

An **angle** AOB consists of two rays R_1 and R_2 with a common vertex O.



Positive Angle
Counterclockwise



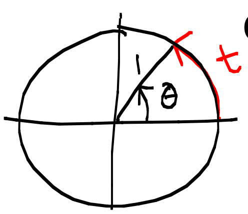
Negative Angle
Clockwise

The **measure of an angle** is the amount of rotation about the vertex required to move R_1 onto R_2.

Units of measurement for an angle:

Degrees - 1 degree is equivalent to rotating the initial side $\frac{1}{360}$ of a complete revolution.

Radians - the amount an angle opens measured along the arc of a circle of radius 1 with its center at the vertex of the angle.



$\boxed{\theta = t}$ on a unit circle (radius = 1)

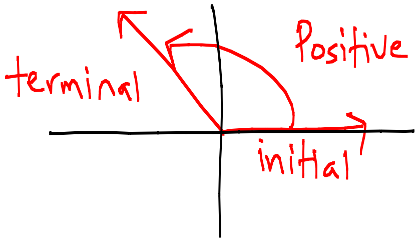
Comparison between degrees and radians $180° = \pi$ radians

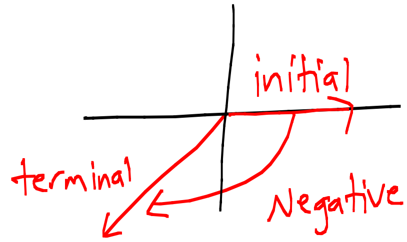
To convert degrees to radians ➞ multiply by $\frac{\pi}{180}$

To convert radians to degress ➞ multiply by $\frac{180}{\pi}$

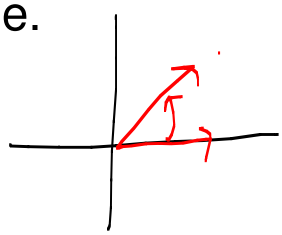
$30° =$ rad $\qquad\qquad$ $\frac{\pi}{5}$ rad $=$ °

An angle is in **standard position** if its vertex is at the origin and its initial side is on the positive $x-$ axis.

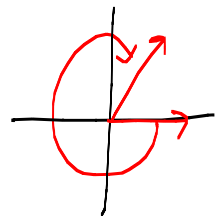




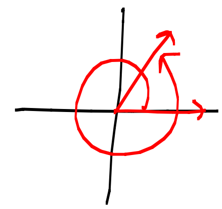
Two angles in standard position are **coterminal** if their sides coincide.



Coterminal to



or



etc.

State two angles that are coterminal with $160°$ (Add or subtract multiples of $360°$)

a positive angle

a negative angle

Find an angle that is coterminal with 1300°.

Assumes θ is in RADIANS!

An angle whose radian measure is θ is subtended by an arc that is the fraction $\dfrac{\theta}{2\pi}$ of the circumference of the circle. Thus in a circle of radius r, the length s of an arc that subtends the angle θ is

$$s = \frac{\theta}{2\pi} \times (circumference\, of\ circle) = \frac{\theta}{2\pi} \times 2\pi r = \theta r$$

Example: a) Find the length of the arc of a circle with $r = 10$ that subtends an angle of 60°.

b) Find the measure of the central angle of a circle subtended by an arc of length 8 if the radius of the circle is 4.

Assumes $\underline{\theta}$ is in $\underline{\text{RADIANS}}$!

The area of a sector of a circle with central angle θ is

$$A = \frac{\theta}{2\pi} \times (area\, of\, circle) = \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2 \theta$$

Find the area of a sector of a circle with central angle of 1 radian and the radius of the circle is 5 inches.

Find the area of the sector of a circle of radius 2 feet formed by an angle of 30°.

A sector of a circle of radius 24 miles has an area of 288 square miles. Find the central angle of the sector.

If a point moves along a circle, there are two ways to describe to motion of the point - linear speed and angular speed.

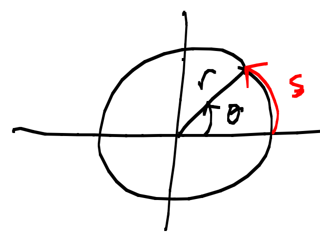
Linear speed is the rate at which the distance traveled is changing, so linear speed is: $\dfrac{\Delta s}{\Delta t} = \dfrac{\text{Change in distance}}{\text{Change in time}}$

Angular speed is the rate at which the central angle θ is changing, so angular speed is: $\dfrac{\Delta \theta}{\Delta t} = \dfrac{\text{Change in } \theta}{\text{Change in time}}$

If a point moves along a circle of radius r and the ray from the center of the circle to the point traverses θ radians in time t. Let $s = r\theta$ be the distance the point travels in time t. Then the speed of the object is given by:

Angular speed $\quad \boldsymbol{\omega = \frac{\theta}{t}}$

Linear Speed $\quad \boldsymbol{v = \frac{s}{t}}$



Recall, $s = r\theta$

If a point moves along a circle of radius r with angular speed ω, then its linear speed v is given by

$$\boldsymbol{v = r\omega}$$

Example: A truck with 48-in. diameter wheels is traveling at 50 mph.

a) Find the angular speed of the wheels in rad/min.
b) How many revolutions per minute do the wheels make?