Applications of Logs and Exponents Video Lecture

Section 5.7 and 5.8

Course Learning Objectives:

1) **Solve appropriate applications of determining compound interest and exponential growth and decay.**
2) **Identify and articulate the significance of graphical components in a mathematical model/application.**

Weekly Learning Objectives:

1) **Find equations of populations that obey the law of uninhibited growth.**
2) **Find equations of populations that obey the law of decay.**
3) **Solve exponential growth and decay applications including compound interest, population growth, and radioactive decay.**
4) **Use Newton's Law of Cooling.**
5) **Solve applications involving logarithms.**

Applications of Logs and Exponentials

Many natural phenomena follow the principle that the rate of growth or decay is proportional to the amount present. A model which will represent this situation is $A = A_0 e^{kt}$, where t represents time, A_0 is the original amount ($t = 0$), A is the amount present at time t and and k is a non-zero constant representing the relative rate of growth or decay.

The growth of money in an interest bearing account is an example of such a phenomenon. As you know, it can be modeled using the formula $A = Pe^{rt}$.

Find the time it takes for an investment to double if the interest rate is 8% compounded continuously.

A model which describes the size of a population at time t is given by $P(t) = P_0 e^{rt}$ where r is the relative rate of growth and P_0 is the initial population.

A bacteria population (initial size 400) triples in $\frac{1}{2}$ hour. Find the model that represents the population.

Determine when the population reaches $1,000,000$.

How many bacteria are present after 75 minutes?

Radio-active decay is analogous to population growth except that the mass decreases rather than increases.

Set up a model to represent the amount of radio-active chemical present if we have 10 grams present initially and the half life is 1000 years.

Determine the amount present after 140 years.

Find how long it takes to have 6.1 grams present.

In 1998 the canvas of a painting attributed to Vermeer (1632 - 1675) was found to contain 99.5% of the carbon-14 that is present in living trees. How long ago was the canvas made? Note: The half life of Carbon-14 is 5730 years.

Newton's law of cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. If T_s represents the temperature of the surroundings, then the temperature of the object at time t is given by $T(t) = T_s + (T_0 - T_s)e^{-kt}$ where k is a positive constant and T_0 is the initial temperature of the object.

A cup of coffee is taken from the microwave at 210 °F and is placed in a room that has a temperature of 68 °F. Find a formula for the temperature of the coffee at time t if the temperature after 5 minutes was $150°$.

What is the temperature of the coffee after 15 minutes?

When has the coffee cooled to 100 °F?

In a filtering system water runs through a series of charcoal filters - each layer removing 30% of the pollutants present. That is, the amount remaining is 70% of the amount present prior to going through each filter. Find a model to represent this situation.

If each filter costs $\$4,000,000$, what would the cost be for a city to remove 95% of the pollutants?

What would the cost be to remove 99% of the pollutants?

What would the cost be to remove 99.9% of the pollutants?

When a physical quantity can vary over a very large range, it is convenient to takes its logarithm in order to have a more manageable set of numbers. The Richter scale which measures the intensity of earthquakes, the pH scale which measures acidity, the decibel scale which measures loudness of sound, and light intensity are examples of situations which use logarithms for this purpose.

The Richter scale uses the model $M = \log \frac{I}{S}$ where I is the intensity of the earthquake and S is the intensity of a "standard" earthquake.

The Northbridge, CA, earthquake had a magnitude of 8.6 on the Richter scale. The earthquake in Japan in 1995 had a Richter scale reading of 7.2. How many times more intense was the California earthquake?