

Composition of Functions Video Lecture

Section 5.1

Course Learning Objectives:

Demonstrate an understanding of the concept of function, and functional attributes such as domain and range. Determine these attributes for a function given its graph and/or its rule; Perform the algebra of functions: sum, difference, product, quotient, and composite.

Weekly Learning Objectives:

- 1) Form the sum, difference, product, and quotient of two functions and the corresponding domain.**
- 2) Form a composite function.**
- 3) Find the domain of a composite function.**

Composition of Functions

Functions may be combined in many ways to form new functions. The most obvious way to combine functions is to use the same operations that are used to combine numbers. i.e. addition, subtraction, etc.

Notation:

$$(f + g)(x) = f(x) + g(x) \quad \text{Domain: Intersection of } D(f) \text{ and } D(g)$$

$$(f - g)(x) = f(x) - g(x) \quad \text{Domain: Intersection of } D(f) \text{ and } D(g)$$

$$(f \cdot g)(x) = f(x) \cdot g(x) \quad \text{Domain: Intersection of } D(f) \text{ and } D(g)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Domain: Intersection of $D(f)$ and $D(g)$ and exclude where $g(x) = 0$

Example: Let $f(x) = \sqrt{1-x}$ and $g(x) = \sqrt{9-x^2}$
Find each of the following:

$$(f \cdot g)(-2) =$$

$$\left(\frac{f}{g}\right)(0) =$$

Domain of $(f-g)(x)$

Domain of $\left(\frac{f}{g}\right)(x)$

Another way of combining two functions together to form a new function is to form the composition of the two functions. The composite function is denoted $(f \circ g)(x)$, (read ***f composed with g*** or ***f of g of x*** or ***the composition of f and g***) and is defined in the following way:

$$(f \circ g)(x) = f(g(x))$$

To find the domain of the composition

$$(f \circ g)(x) = f(g(x)),$$

first find a formula for the composition. Then take the intersection of the domain of the formula with the domain of $g(x)$.

If $f(x) = \frac{x+1}{x-2}$ and $g(x) = \frac{x-3}{2x+5}$,

find the domain of each of the following:

a) $f(g(x))$

b) $(g \circ f)(x)$

Find $f(x)$ and $g(x)$ such that $f(g(x)) = \frac{1}{\sqrt{x+3}}$