Dividing Polynomials Video Lecture

Section 4.2

Course Learning Objectives:
Demonstrate appropriate manipulation of rational expression. (Review from Math 940)

Weekly Learning Objectives:

1) Use long division to divide polynomials.
2) Use synthetic division to divide polynomials.
3) Use the division algorithm.
4) Use the remainder and factor theorems.

Dividing Polynomials

Review of Long Division:

$$x-2\overline{)12x^3-5x^2-14x+1}$$

$$x-1\overline{)3x^4-x^2+1}$$

Review of Synthetic Division: (can only be used when dividing by a LINEAR factor!)

$$(12x^3-5x^2-14x+1)\div(x-2)$$

$$x-1\,\overline{)\,3x^4-x^2+1}$$

The Division Algorithm summarizes the results of any polynomial division.

If P(x) and D(x) are polynomials, with D(x) not equal to zero, then there exist unique polynomials Q(x) and R(x) such that

$$P(x) = D(x) \cdot Q(x) + R(x)$$

where R(x) is either 0 or of degree less than the degree of D(x).

Show how the Division Algorithm applies to the last example.

Note: If the divisor in the Division Algorithm is of the form x - c for some real number c then the remainder must be a constant. The Division Algorithm could be restated in this situation as:

Let x = c in the above equation:

Remainder Theorem: If the polynomial P(x) is divided by x - c, then the remainder is the value P(c).

What is the remainder when dividing $x^{14} - 3x^{12} + x^{10} - 4x + 5$ **by** $x - 1$?

Use synthetic division to solve: $(x^3 - x^2 + 5x - 2) \div (x - 2)$

What other information do we get because of the Remainder Theorem?

Factor Theorem: P(c) = 0 if and only if x - c is a factor of P(x).

Proof:

Is $x-1$ a factor of $x^{12}-3x^{10}+5x^{8}+2x^{5}-4x^{2}-1$?

State any theorems you are using.

Reminder: For every zero there is an associated factor.

Find a polynomial function of degree 4 with zeros 3, -2, 0 and 1.