Increasing, Decreasing, Even, Odd, AROC Video Lecture

Section 2.3

Course Learning Objectives:

1) Demonstrate an understanding of functional attributes such as increasing and decreasing. Determine these attributes for a function given its graph and/or its rule.
2) Identify and articulate the significance of graphical components such as intervals of increase or decrease in a mathematical model/application.

Weekly Learning Objectives:

1) Use a graph to determine where a function is increasing, decreasing, or constant.
2) Use a graph to locate local maxima and local minima.
3) Use a graphing utility to approximate local maxima and local minima and to determine where a function is increasing or decreasing.
4) Identify even and odd functions from the equation.
5) Find the average rate of change of a function.
6) Find the simplified difference quotient of a function.
7) Use a difference quotient to find an average rate of change.

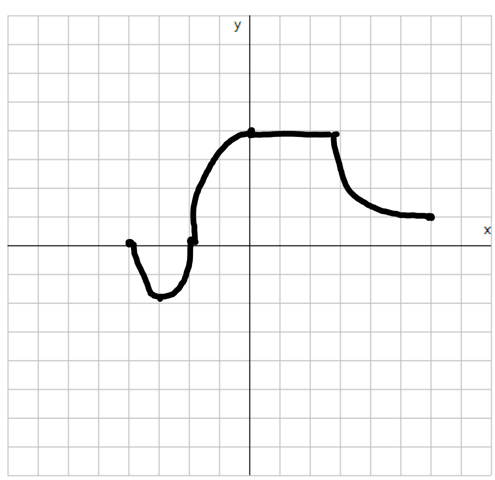
Increasing, Decreasing, Even, Odd and Average Rates of Change

Increasing Functions: A function is increasing on an open interval I if, for any choice of a and b in I, with a < b, then f(a) < f(b). Informally, this means the graph of f(x) rises from left to right.

Decreasing Functions: A function is decreasing on an open interval I if, for any choice of a and b in I, with a < b, then f(a) > f(b). Informally, this means the graph of f(x) falls from left to right.

A function is **constant** on an open interval I if, for all choices of x in I, the values of f(x) are equal. The graph is a constant height (horizontal) as you go left to right.

Determine where the following graph is increasing, decreasing and constant.



Use your graphing calculator to determine where the function is increasing and decreasing:

$$f(x) = -x^4 + x^3 + 3x^2 - x + 18$$

Notice on the last two examples, that when a graph changed from increasing to decreasing it had a maximum. When the graph changed from decreasing to increasing it had a minimum. These are called **local maximums** or **local minimums**.

If f has a local maximum at c, then the value of f at c is greater than the values of f near c. (Locally near c, f(c) is the largest y-value.)

If f has a local minimum at c, then the value of f at c is less than the values of f near c. (Locally near c, f(c) is the smallest y-value.)

Use your graphing calculator to determine where f(x) is increasing and decreasing. Also, determine where f(x) has a local maximum and local minimum.

$$f(x) = 6x^3 - 12x + 5$$

Recall the following definitions:

EVEN: symmetry about the y-axis **Test: Replace x with -x**

ODD: symmetry about the origin **Test: Replace x with -x and y with -y**

These definitions are equivalent to the following function definitions:

EVEN: f is even if f(-x) = f(x)

ODD: f is odd if f(-x) = -f(x)

Show algebraically whether the following functions are even, odd or neither.

$f(x) = 5x^3 - x$

$f(x) = \frac{x}{x^2 - 1}$

$f(x) = x^2 + |x|$

$f(x) = \sqrt[3]{x} + 1$

Suppose the graph of f(x) contains the point (4, -1). What point is guaranteed to be on:

a) the graph of f if f is even?

b) the graph of f if f is odd?

c) the graph of y = 2f(x) + 3?

Average Rate of Change:

The average rate of change in a function f(x) from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ is defined to be:

$$\text{A. R. O. C.} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\Delta f}{\Delta x}$$

The average rate of change also represents the slope of the **secant line** between the two points.

Find the average rate of change of $f(x) = 3x^2 - 2x + 3$ from -2 to 1.

Find the equation of the secant line between (-2, f(-2)) and (1, f(1)).

An alternate form to calculate the Average Rate of Change with a fixed starting point:

Given f(x) on the interval [a, a+h],

$$\text{A.R.O.C.} = \frac{\Delta f}{\Delta x} = \frac{f(a+h) - f(a)}{h}$$

This is called the **DIFFERENCE QUOTIENT** of f(x).

Find the simplified difference quotient for the following functions:

$f(x) = 2x^2 - x + 1$

$f(x) = \sqrt{x} + 1$

$f(x) = \frac{1}{x-2}$

For the last difference quotient, find the average rate of change:

a) when $a = 4$ and $h = .5$

b) over the interval $[3, 3.01]$