

Inverse Functions Video Lecture

Section 5.2

Course Learning Objectives:

Demonstrate an understanding of the concept of function. Determine these attributes for a function given its graph and/or its rule; Perform the algebra of functions including finding an inverse.

Weekly Learning Objectives:

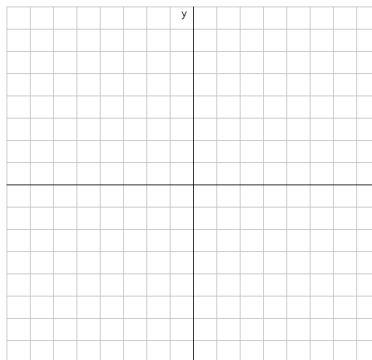
- 1) Determine whether a function is one-to-one.**
- 2) Use the Horizontal Line Test.**
- 3) Determine the inverse of a function by a map or a set of ordered pairs.**
- 4) Obtain the graph of the inverse function from the graph of the function.**
- 5) Find the inverse of a function defined by an equation.**
- 6) Verify that two functions are inverses of each other.**

Inverse Functions

Consider the relation $f : \{(-2, 1), (5, 4), (3, 1), (6, 2)\}$. Is f a function?

Draw a mapping:

Draw a graph:

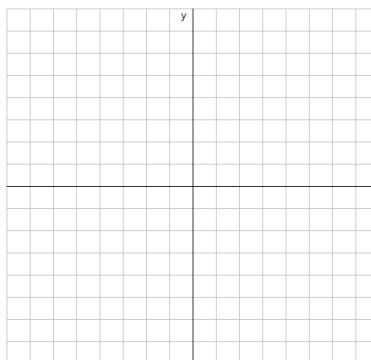


Now reverse the correspondence (find the inverse):

List:

Draw a mapping:

Draw a graph:

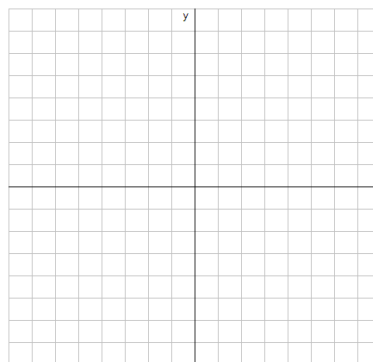


Do we obtain another function?

Let $f(x) = x^2$

Sample mapping:

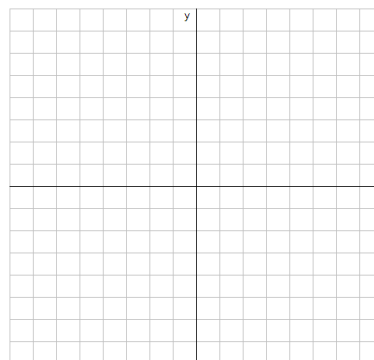
Draw Graph:



Reverse the correspondence (find the inverse):

Sample mapping:

Draw Graph:



Do we get another function? If not, what could we do to guarantee another function?

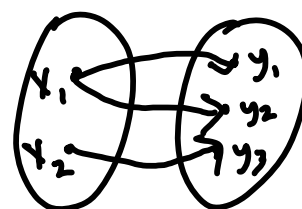
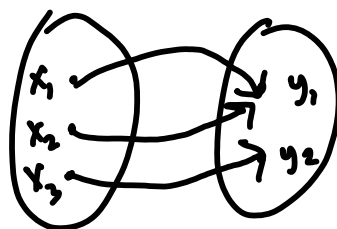
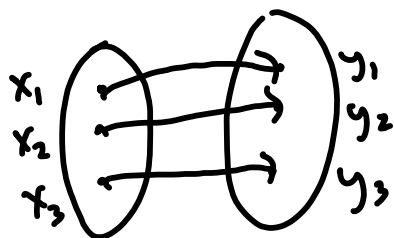
In order for a relation to be a function, arguments (x-values or inputs) cannot have more than one image (y-value or output). (No two ordered pairs can have the same first element).

In order for a function to be "reversible" and still yield a function, images (y-values or outputs) cannot have more than one argument (x-value or input). No two ordered pairs can have the same second element. Such a function is said to be one-to-one.

Definition: $f(x)$ is one-to-one if and only if when $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$

or

$f(x)$ is one-to-one if and only if when $f(x_1) = f(x_2)$ then $x_1 = x_2$.



Determine whether the following are one-to-one:

$$f : \{ (1, 3), (4, 2), (-1, 3) \}$$

$$h : \{ (-2, 4), (2, 6), (1, 1) \}$$

$$g(x) = x^2 - 3x - 4$$

Horizontal Line Test: If every horizontal line intersects the graph of a function f in at most one point, then f is a one-to-one function.

A function which will undo the operations given in a function $f(x)$ is called the inverse of $f(x)$, and is denoted by $f^{-1}(x)$.

In order to have an inverse, f must be one-to-one.

If $f(x)$ includes the ordered pairs (x,y) , then the inverse function, $f^{-1}(x)$, includes the ordered pairs (y,x) (just swap x and y). Since the inverse is formed by interchanging x and y , the domain of $f(x)$ becomes the range of $f^{-1}(x)$ and the range of $f(x)$ becomes the domain of $f^{-1}(x)$.

The inverse of a function satisfies the condition that

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x.$$

Given that $f(x) = \frac{x+1}{3x-2}$ show that $g(x) = \frac{2x+1}{3x-1}$ is the inverse of $f(x)$.

To find the inverse of a function:

After determining that a function f is one-to one, find its inverse by:

1. Replacing $f(x)$ with y
2. Interchanging x and y
3. Solving for y in terms of x
4. Replacing y with $f^{-1}(x)$ notation.

Step 2 above is sufficient for finding an inverse if the function is not expressed using standard functional notation. Finding the inverse of a function using the above procedure guarantees the formula is solved for $f^{-1}(x)$. It is not necessary to verify that the two functions are inverses of each other.

Verify that the following functions are one-to-one, then find the inverse.

$$f: \{(1, 3), (2, 4), (0, -3)\}$$

$$g(x) = 2x + 1$$

$$h(x) = \sqrt{x-1}$$

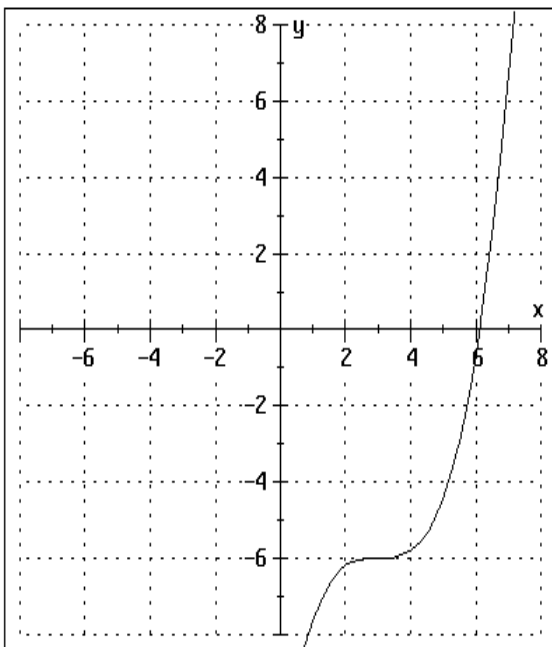
$$p(x) = -x^2 + 1$$

$$q(x) = \frac{4x-1}{3x+2}$$

$$j(x) = x^2 + x, \quad x \geq 0$$

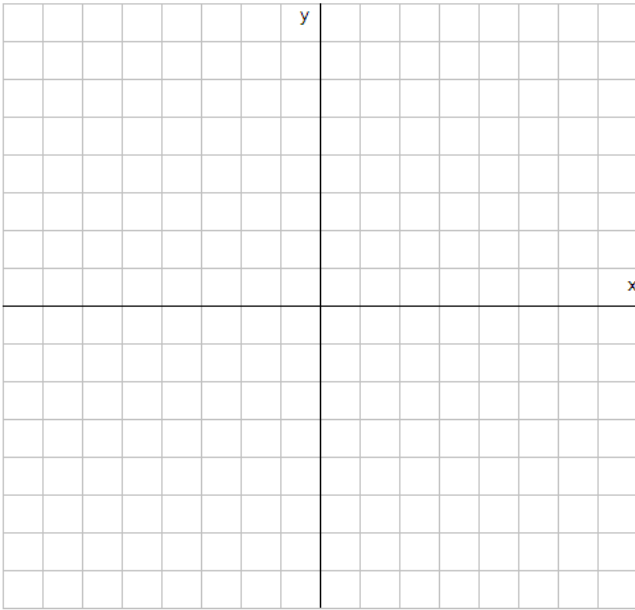
If $f(3) = 6$, what is $f^{-1}(6) = ?$

The graph below is $f(x)$. Find $f^{-1}(6)$.

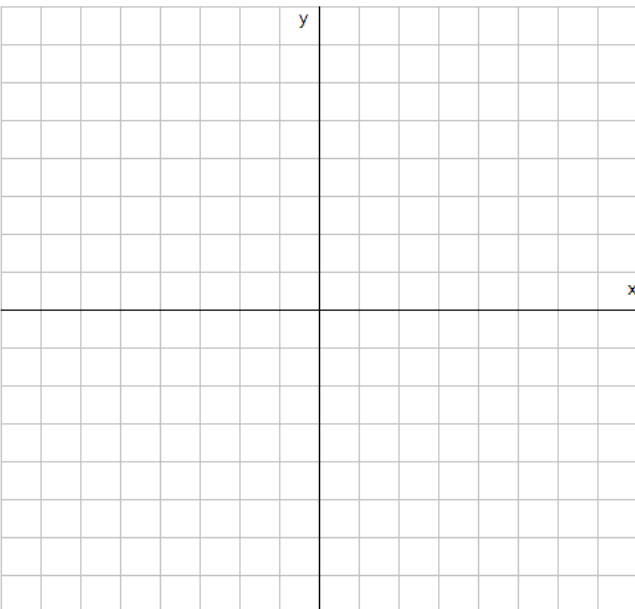


Graph $f(x)$ and $f^{-1}(x)$ on the same set of axes:

$$f(x) = 2x - 3$$



$$f(x) = -\sqrt{x-1}$$



The graph of a function f and the graph of its inverse $f^{-1}(x)$ are symmetric with respect to the line $y = x$.

The graph below is $f(x)$. Draw the graph of the inverse.

