# Inverse Functions Video Lecture

## Section 5.2

**Course Learning Objectives:**
**Demonstrate an understanding of the concept of function. Determine these attributes for a function given its graph and/or its rule; Perform the algebra of functions including finding an inverse.**

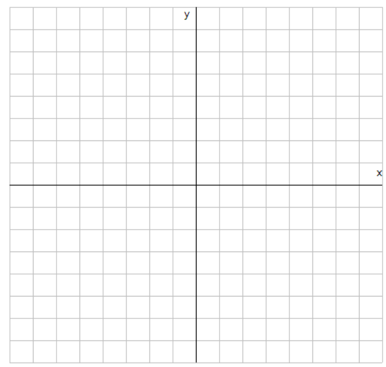
**Weekly Learning Objectives:**

1) **Determine whether a function is one-to-one.**
2) **Use the Horizontal Line Test.**
3) **Determine the inverse of a function by a map or a set of ordered pairs.**
4) **Obtain the graph of the inverse function from the graph of the function.**
5) **Find the inverse of a function defined by an equation.**
6) **Verify that two functions are inverses of each other.**

# Inverse Functions

**Consider the relation f : {(-2, 1), (5, 4), (3, 1), (6, 2)}.  Is f a function?**

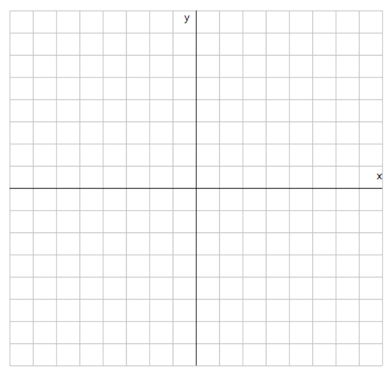
**Draw a mapping:** **Draw a graph:**




**Now reverse the correspondence (find the inverse):**

**List:**

**Draw a mapping:** **Draw a graph:**

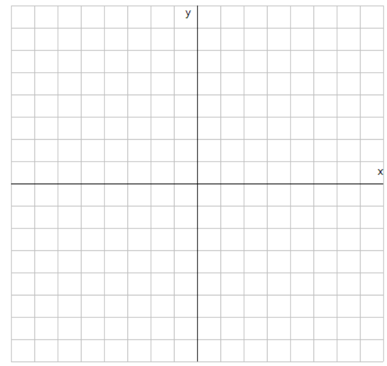



**Do we obtain another function?**

**Let f(x) =** $x^2$

**Sample mapping:**

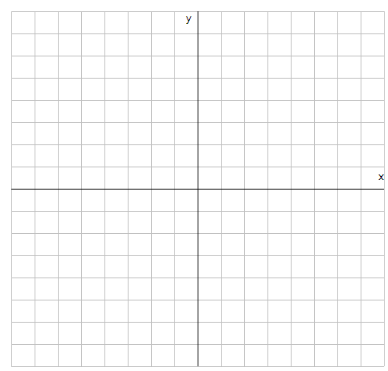
**Draw Graph:**




**Reverse the correspondence (find the inverse):**

**Sample mapping:**

**Draw Graph:**




**Do we get another function? If not, what could we do to guarantee another function?**

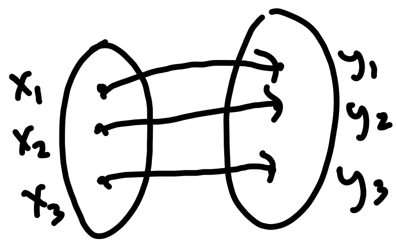
**In order for a relation to be a function, arguments (x-values or inputs) cannot have more than one image (y-value or output). (No two ordered pairs can have the same first element).**

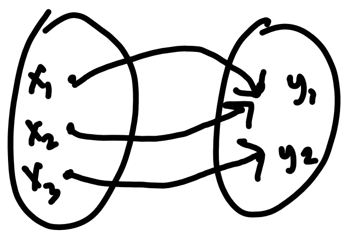
**In order for a function to be “reversible" and still yield a function, images (y-values or outputs) cannot have more than one argument (x-value or input). No two ordered pairs can have the same second element. Such a function is said to be one-to-one.**

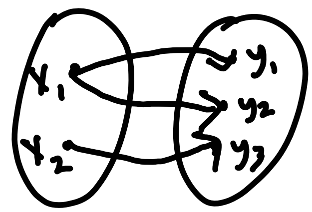
**Definition: f(x) is one-to-one if and only if when** $x_1 \neq x_2$ **then** $f(x_1) \neq f(x_2)$

**or**

**f(x) is one-to-one if and only if when** $f(x_1) = f(x_2)$ **then** $x_1 = x_2$.




**Determine whether the following are one-to-one:**

**f : { (1, 3), (4, 2), (-1, 3)}**

**h: {(-2, 4), (2, 6), (1, 1)}**

$g(x) = x^2 - 3x - 4$

**Horizontal Line Test: If every horizontal line intersects the graph of a function f in at most one point, then f is a one-to-one function.**

**A function which will <u>undo</u> the operations given in a function f(x) is called the inverse of f(x), and is denoted by** $f^{-1}(x)$.

**<u>In order to have an inverse, f must be one-to-one.</u>**

**If f(x) includes the ordered pairs (x,y), then the inverse function,** $f^{-1}(x)$ **, includes the ordered pairs (y,x) (just swap x and y). Since the inverse is formed by interchanging x and y, the domain of f(x) becomes the range of** $f^{-1}(x)$ **and the range of f(x) becomes the domain of** $f^{-1}(x)$**.**

**The inverse of a function satisfies the condition that**

$f(f^{-1}(x)) = x$ **and** $f^{-1}(f(x)) = x$.

**Given that f(x) =** $\frac{x+1}{3x-2}$ **show that g(x) =** $\frac{2x+1}{3x-1}$ **is the inverse of f(x).**

**To find the inverse of a function:**

**After determining that a function f is one-to one, find its inverse by:**

1. **Replacing f(x) with y**
2. **Interchanging x and y**
3. **Solving for y in terms of x**
4. **Replacing y with** $f^{-1}(x)$ **notation.**

**Step 2 above is sufficient for finding an inverse if the function is not expressed using standard functional notation. Finding the inverse of a function using the above procedure guarantees the formula** $f^{-1}(x)$ **is solved for**
**It is not necessary to verify that the two functions are inverses of each other.**

**Verify that the following functions are one-to-one, then find the inverse.**

**f: {(1, 3), (2, 4), (0, -3)}**

**g(x) = 2x + 1**

**h(x) =** $\sqrt{x-1}$

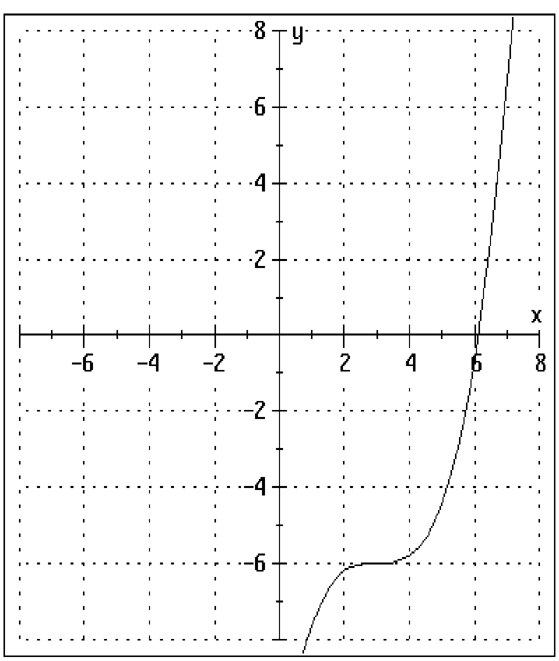
**p(x) =** $-x^2+1$

**q(x) =** $\dfrac{4x-1}{3x+2}$

**j(x) =** $x^2+x \;,\; x \geq 0$

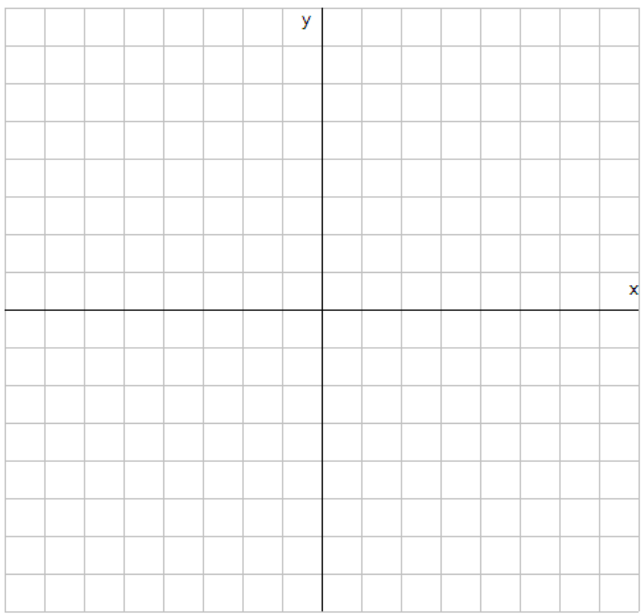
**If f(3) = 6, what is** $f^{-1}(6) = ?$

**The graph below is f(x). Find** $f^{-1}(6)$.

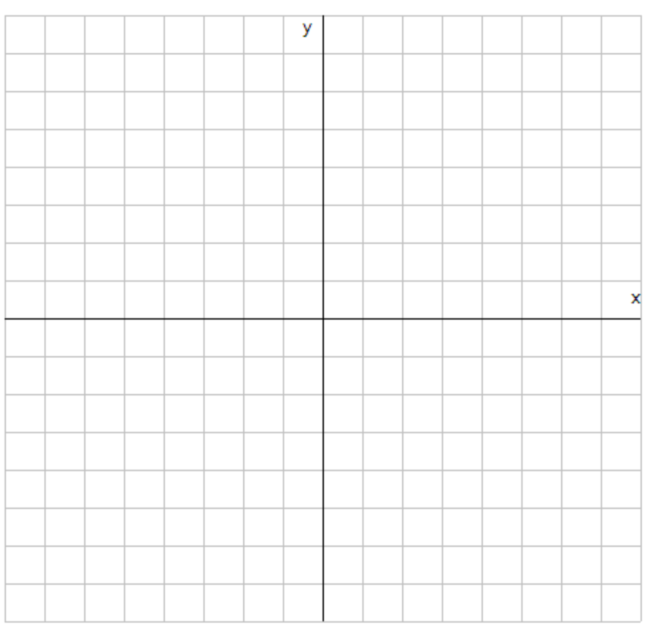


**Graph f(x) and $f^{-1}(x)$ on the same set of axes:**

**f(x) = 2x - 3**




**f(x) =** $-\sqrt{x-1}$



**The graph of a function f and the graph of its inverse $f^{-1}(x)$ are symmetric with respect to the line y = x.**

**The graph below is f(x).  Draw the graph of the inverse.**