Inverse Trig Functions

Sections 7.1 and 7.2

Course Learning Objectives:

1) **Demonstrate an understanding of trigonometric functions and their applications.**
2) **Verify identities and solve trigonometric equations.**

Weekly Learning Objectives:

1) **Find the exact value of an inverse sine, cosine, or tangent function.**
2) **Use the appropriate domain and range defined by the inverse sine, cosine and tangent functions.**
3) **Find an approximate value of an inverse trigonometric function.**
4) **Use properties of inverse functions to find exact values of certain composite functions.**
5) **Find the inverse function of a trigonometric function.**
6) **Solve equations involving inverse trigonometric functions.**
7) **Find the exact value of expressions involving the inverse sine, cosine, and tangent functions.**
8) **Use sum and difference formulas involving inverse trigonometric functions.**
9) **Know the definitions of the inverse secant, cosecant, and**

Inverse Trigonometric Functions

Facts about Inverse functions:

If f is a one-to-one function with Domain A and Range B, then

f^{-1} is the inverse function with Domain B and Range A.

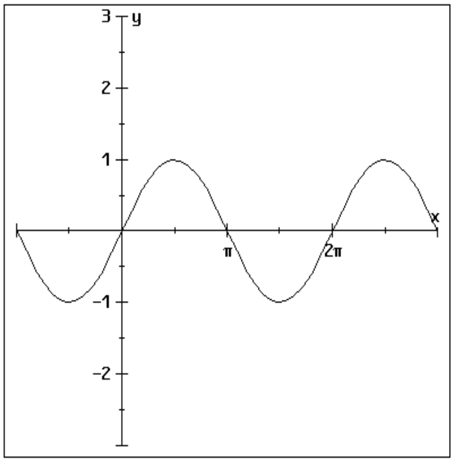
(swap x's and y's)

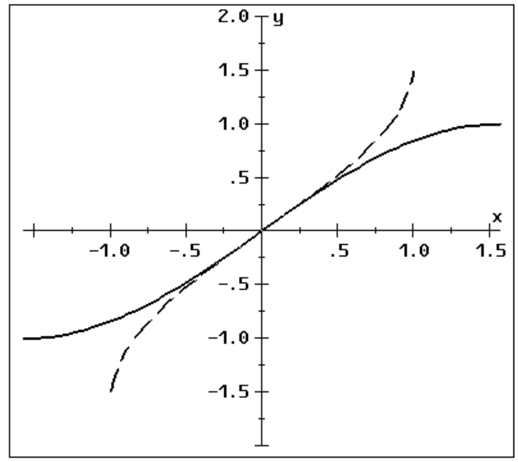
One-to-one: For every x there is exactly one and only y.

A function is one-to-one if it passes the VLT and HLT.

Is the sine function 1-1?

We can find the inverse if we restrict the domain in such a way so that the portion of the function that we consider is 1-1.





Def: The inverse sine function is the function $\sin^{-1}(x)$ with domain and range defined by

$$\boldsymbol{\sin^{-1}(x) = y \;\Leftrightarrow \sin y = x}.$$

The inverse sine function is also called the arcsine function and denoted arcsin x.

$$\sin(\sin^{-1}(x)) = \qquad\qquad -1 \le x \le 1$$

$$\sin^{-1}(\sin(x)) = \qquad\qquad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

Evaluate each of the following:

$\sin^{-1}\left(\frac{1}{2}\right)$ $\qquad$ $\sin^{-1}\left(-\frac{1}{2}\right)$ $\qquad$ $\sin^{-1}\left(\frac{3}{2}\right)$ $\qquad$ $\sin^{-1}\left(\frac{1}{3}\right)$

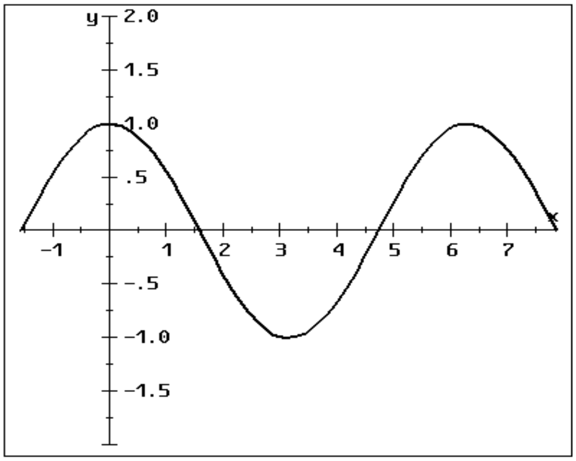
$\sin^{-1}\left(\sin\frac{\pi}{6}\right)$ $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ $\sin^{-1}\left(\sin\frac{11\pi}{6}\right)$

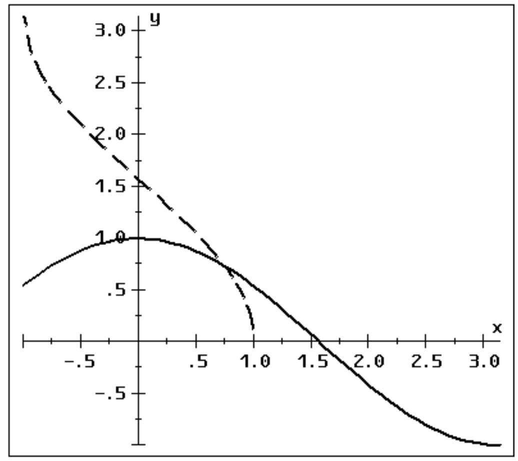
$\sin\left(\sin^{-1}\frac{1}{2}\right)$ $\sin\left(\sin^{-1}\frac{3}{4}\right)$

$\cos\left(\sin^{-1}\frac{2}{7}\right)$ $\tan\left(\sin^{-1}-\frac{3}{4}\right)$

Is the cosine function 1-1?

We can find the inverse if we restrict the domain in such a way so that the portion of the function that we consider is 1-1.





Def: The inverse cosine function is the function $\cos^{-1}(x)$ with domain and range defined by

$$\cos^{-1}(\boldsymbol{x}) = \boldsymbol{y} \;\Leftrightarrow \cos \boldsymbol{y} = \boldsymbol{x}.$$

The inverse cosine function is also called the arccosine function and denoted arccos x.

$\cos(\cos^{-1}(x)) =$ $\qquad -1 \leq x \leq 1$

$\cos^{-1}(\cos(x)) =$ $\qquad 0 \leq x \leq \pi$

Evaluate each of the following:

$\cos^{-1}\left(\frac{1}{2}\right)$ $\qquad \cos^{-1}\left(-\frac{1}{2}\right)$ $\qquad \cos^{-1}\left(\frac{3}{2}\right)$ $\qquad \cos^{-1}\left(\frac{1}{3}\right)$

$$\cos^{-1}\left(\cos\frac{\pi}{6}\right) \qquad \cos^{-1}\left(\cos\frac{5\pi}{4}\right) \qquad \cos^{-1}\left(\cos\frac{11\pi}{6}\right)$$

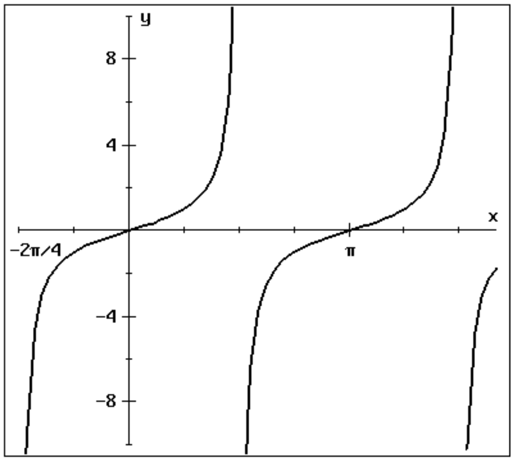
$$\cos\left(\cos^{-1}\frac{1}{2}\right) \qquad \cos\left(\cos^{-1}\frac{3}{4}\right)$$

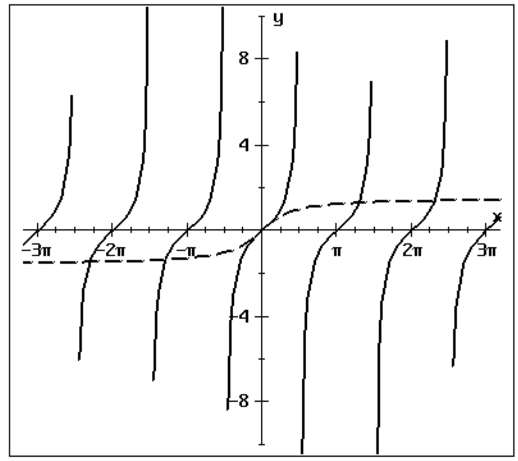
$$\cos\left(\cos^{-1}\frac{2}{7}+\sin^{-1}\left(-\frac{3}{4}\right)\right) \qquad \tan\left(\cos^{-1}-\frac{3}{4}\right)$$

$$\sin\left(2\cos^{-1}x\right)$$

Is the tangent function 1-1?

We can find the inverse if we restrict the domain in such a way so that the portion of the function that we consider is 1-1.





Def: The inverse tangent function is the function $\tan^{-1}(x)$ with domain and range defined by

$$\tan^{-1}(\boldsymbol{x}) = \boldsymbol{y} \;\Leftrightarrow \tan \boldsymbol{y} = \boldsymbol{x}.$$

The inverse tangent function is also called the arctangent function and denoted arctan x.

$\tan(\tan^{-1}(x)) =$ $\qquad\qquad$ $x \in R$

$\tan^{-1}(\tan(x)) =$ $\qquad\qquad$ $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

Evaluate each of the following:

$$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) \qquad \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) \qquad \tan^{-1}\left(\frac{3}{2}\right) \qquad \tan^{-1}\left(\frac{1}{3}\right)$$

$\tan^{-1}\left(\tan\frac{\pi}{6}\right)$ $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$ $\tan^{-1}\left(\tan\frac{11\pi}{6}\right)$

$\tan\left(\tan^{-1}\left(\frac{1}{2}\right)\right)$ $\tan\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$

$\tan\left(\cos^{-1}\left(\frac{2}{7}\right)\right)$ $\tan\left(\tan^{-1}\left(-\frac{3}{4}\right)+\cos^{-1}\left(\frac{2}{3}\right)\right)$

$\sin\left(2\tan^{-1}x\right)$

$$\sin\left(\cos^{-1}\left(-\frac{2}{3}\right)+\arcsin\left(\frac{1}{4}\right)\right)$$

Solve the equation: $3\sin^{-1}x = \pi$

The remaining three trigonometric functions also have inverses if we appropriately restrict their domains. You will not be held responsible for the specific restrictions and thus the domains for $\csc^{-1}(x)$, $\sec^{-1}(x)$, and $\cot^{-1}(x)$.

Use your calculator to find each of the following in terms of radians and degrees:

$\cot^{-1}(5)$ $\sec^{-1}(-3)$ $\csc^{-1}(\pi)$