Linear Programming Video Lecture

Section 11.8

Course Learning Objectives:
Solve appropriate applications. Identify and articulate the significance of graphical components in a mathematical model/application.

Weekly Learning Objectives:

1) **Set up a linear programming problem.**
2) **Solve a linear programming problem.**

Linear Programming

Linear programming is a modeling technique used to determine the optimal allocation of resources. It was developed during World War II to allocate troops and materials in different areas of Europe. Linear programming can be applied to very complex problems involving thousands of variables and thousands of equations. Airline schedules, telephone connections, and the trucking industry are just a few examples of industries making use of linear programming.

We will look some simple examples where we can use a graphical approach by graphing a system of linear inequalities. This can be done as long as the situation can be described with only two variables. In complex examples matrices and matrix operations as used to determine the optimal solution.

Example 1:

A retired couple has up to $25,000 to invest. As their financial advisor, you recommend that they place at least $15,000 in Treasury bills yielding 6% and at most $5000 in corporate bonds yielding 9%. How much money should be placed in each investment so that income is maximized?

The process can be summarized in the following four steps.

1) Write an expression for the quantity to be maximized (or minimized). This expression is the objective function.
2) Write all the constraints as a system of linear inequalitites and graph the system.
3) List the corner points of the graph of the feasible points.
4) List the corresponding values of the objective function at each corner point. The largest (or smallest) of these is the solution.

Example 2:

Minimize the expression

$$z = 2x + 3y$$

subject to the constraints

$y \leq 5$ $x \leq 6$ $x + y \geq 2$ $x \geq 0$ $y \geq 0$

Example 3:

At the end if every month, after filling orders for its regular customers, a coffee company has some pure Colombian coffee and some special-blend coffee remaining. The practice of the company has been to package a mixture of the two coffees into 1 pound packages as follows: a low-grade mixture containing 4 oz of Colombian coffee and 12 oz of special-blend coffee and a high-grade mixture containing 8 oz of Colombian and 8 oz of special-blend. A profit of $0.30 per package is made on the low-grade mixture, whereas a profit of $0.40 per package is made on the high-grade mixture. This month, 120 lb of special-blend coffee and 100 lb of Colombian coffee remain. How many packages of each mixture should be prepared to maximize profit? Assume that all pakages prepared can be sold.