Logarithmic Functions Video Lecture

Section 5.4

Course Learning Objectives:

1) Graph logarithmic functions and use such graphs to solve applied problems and to understand the significance of attributes of the graph to such applied problems.
2) Identify and articulate the significance of graphical components in a mathematical model/application.

Weekly Learning Objectives:

1) Change exponential statements to logarithmic statements and logarithmic statements to exponential statements.
2) Evaluate logarithmic expressions.
3) Determine the domain of a logarithmic function.
4) Graph logarithmic functions with transformations.
5) Evaluate logarithms whose base is neither 10 nor e.
6) Graph logarithmic functions whose base is neither 10 nor e.

Logarithmic Functions

Recall that the exponential function is 1-1 and therefore has an inverse function. Remember we find the inverse of a function by swapping x and y and solving for the new y. Solving this equation for y gives rise to the definition of the logarithmic function.

The logarithmic function with base a, denoted by $\log_a x$, is defined by $y = \log_a x \;\Leftrightarrow a^y = x$. In other words $\log_a x$ is the EXPONENT needed.

Evaluate each of the following:

1. $\log_2 16$ **2**. $\log_2 \frac{1}{8}$ **3**. $\log_2 1$

4. $\log_4 8$ **5**. $\log_2 \left(-\frac{1}{2}\right)$

6. Solve for x : $\log_x 25 = 2$ **7**. Solve for x : $\log_5 x = 4$

The two logarithmic functions used most frequently are the common log function $f(x) = \log_{10} x$ which we will write as $f(x) = \log x$ and the natural log function $g(x) = \log_e x$ which we will write as $g(x) = \ln x$.

8. Write in exponential form: $\ln(x-1) = 4$

9. Write in logarithm form: $e^{x+1} = .5$

Characteristics of the graph of the logarithmic function $f(x) = \log_a x$

1. The graph of $f(x) = \log_a x$ will always contain the point $(1, 0)$.

2. The graph will always contain the points $(a, 1)$ and $\left(\frac{1}{a}, -1\right)$

3. When $a > 1$ the graph is increasing. When $0 < a < 1$ the graph is decreasing.

4. The graph is asymptotic to the negative y-axis when $a > 1$. $(x \to 0^+, f(x) \to -\infty)$. The graph is asymptotic to the positive y-axis when $0 < a < 1$. $(x \to \infty, f(x) \to \infty)$

5. The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.

6. The function is one-to-one.

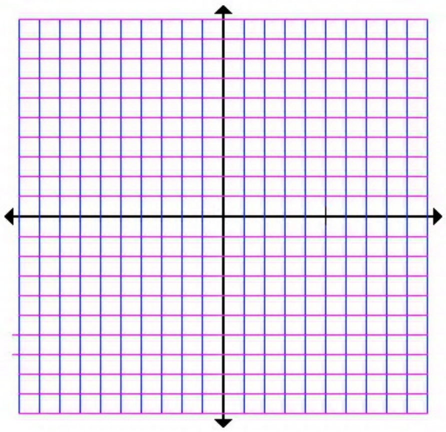
7. All previously discussed transformations apply to the graph of an logarithmic function.

To graph $f(x) = \log_a x$, make a table with the following inputs:

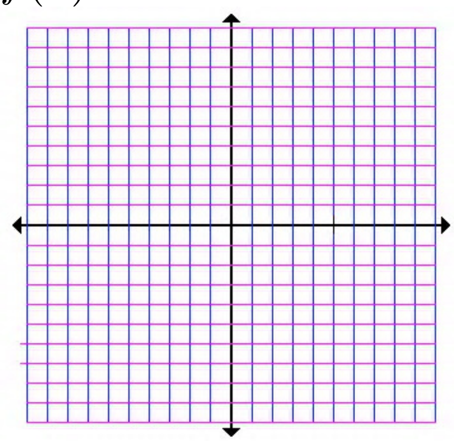
x	y
a	
$\frac{1}{a}$	
1	

Sketch the graph of each of the following indicating the transformations applied to the basic function.

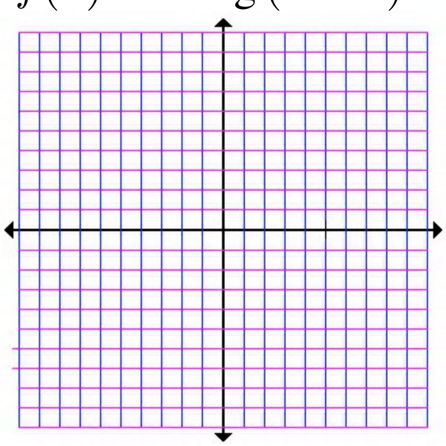
10. $h(x) = \log_4 x + 3$



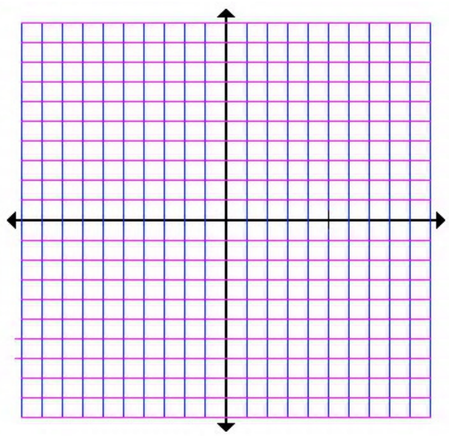
11. $f(x) = 2 \ln x - 1$



12. $f(x) = -\log(x - 1)$



13. $g(x) = 2\log_6(x+1)$



14. List the transformations and find the vertical asymptote for:

$$f(x) = \ln(2-3x)$$

15. Find the domain of $f(x) = \ln\left(x^2 - 3x\right)$

Special properties of the logarithm function

i) $\log_a 1 = 0$

ii) $\log_a a =$

iii) $\log_a a^x =$

iv) $a^{\log_a x} =$