Matrix Algebra I Video Lecture

Section 11.4

Course Learning Objectives:
Perform the algebra of matrices, find inverses of matrices, and solve matrix equations.

Weekly Learning Objectives:

1) Find the sum and difference of two matrices.
2) Find scalar multiples of a matrix.
3) Find the product of two matrices.
4) Solve applications involving matrices.

Matrix Algebra Part I

In this section we look at matrices in a more complete fashion. We have used matrices to make it possible to use technology to solve systems of linear equations. They have many other uses in mathematics and sciences. Matrices can be added, subtracted, multiplied, cubed, etc. We will learn how to perform some of these operations in this section. Preliminarily we need to give some definitions of terms related to matrices.

Dimension:

Entry:

Equality:

Addition:

Subtraction:

Example 1. Given the following matrices: A = $\begin{bmatrix} -1 & 2 \\ 3 & -2 \\ 4 & 0 \end{bmatrix}$

B = $\begin{bmatrix} 3 & -1 & 0 \\ 2 & -2 & 3 \\ 1 & 6 & 2 \end{bmatrix}$ C = $\begin{bmatrix} 2 & 4 \\ 3 & 1 \\ -2 & 2 \end{bmatrix}$ D = $\begin{bmatrix} 2 & -2 & 4 \\ 3 & 6 & 2 \\ -2 & 3 & 1 \end{bmatrix}$

Compute A + C

B – D

D – C

What do you think is the value of 3B?

If A is a matrix and c is a real number, then the scalar product cA is the matrix obtained by multiplying each entry of A by c. This is referred to as scalar multiplication.

Compute 2A – 3C 4D + 2C

Matrix Multiplication:

This becomes a little more involved since it is NOT JUST MULTIPLYING CORRESPONDING ENTRIES!!!! Let A be a matrix of dimension $m \times n$ and B be a matrix of dimension $n \times p$.

1) The dimension of the product matrix will be $m \times p$.
2) The value the c_{ij} (the value of the entry in the product matrix that is in row i and column j) is
   $c_{ij} =$ (1st value in row i of A)(1st value in column j of B) + (2nd value in row i of A)(2nd value in column j of B) +
   . . . . . +
   . . . . . +
   . . . . . +
   (last value in row i of A)(last value in column j of B)
   [This is called the inner product of row i and column j.]

Example 2. If A = $\begin{bmatrix} 3 & -1 & 0 \\ 2 & -2 & 3 \\ 1 & 6 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 \\ 3 & -2 \\ 4 & 0 \end{bmatrix}$

Find AB

Example 3.

Abbot and Costello's stock holdings are given by the matrix

$$A = \begin{array}{c} \text{Abbot} \\ \text{Costello} \end{array} \overset{\begin{array}{cccc} \text{BAC} & \text{GM} & \text{IBM} & \text{TRW} \end{array}}{\begin{bmatrix} 200 & 300 & 100 & 200 \\ 100 & 200 & 400 & 0 \end{bmatrix}}$$

At the close of trading on a certain day, the prices (in dollars per share) of the stocks are given by the matrix

$B = \begin{bmatrix} 54 \\ 48 \\ 98 \\ 82 \end{bmatrix}$ where the values are for BAC, GM, IBM, TRW respectively.

Find AB and interpret what the entries of AB represent!

Find BA.

Example 4. The total output of loudspeaker systems of the Acrosonic Company in their three production facilities for <u>May</u> is given by matrix A = $\begin{bmatrix} 320 & 280 & 460 & 280 \\ 480 & 360 & 580 & 0 \\ 540 & 420 & 200 & 880 \end{bmatrix}$ where the rows could be labeled Location I, Location II, & Location III; and the columns could be labeled Model A, Model B, Model C, & Model D for the four different types of loudspeakers.

The total output of the four loudspeaker systems in their three production facilities for <u>June</u> is given by matrix

B = $\begin{bmatrix} 210 & 180 & 330 & 180 \\ 400 & 300 & 450 & 40 \\ 420 & 280 & 180 & 740 \end{bmatrix}$ (labeled same as matrix A.)

The unit <u>production costs</u> are represented by

C = $\begin{bmatrix} 120 \\ 180 \\ 260 \\ 500 \end{bmatrix}$ where the rows represent the four models.

The unit <u>selling prices</u> are represented by

D = $\begin{bmatrix} 160 \\ 250 \\ 350 \\ 700 \end{bmatrix}$ where the rows represent the four models.

Compute the following matrices AND explain the meaning of the entries in each matrix.

AC AD

A(D – C) B(D – C) (A + B)(D – C)

Example 5.

Let A = $\begin{bmatrix} 2 & -4 & 1 \\ -3 & 6 & -5 \\ 1 & -3 & 7 \end{bmatrix}$ X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ B = $\begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$

What would the matrix equation AX = B represent?
[Perform the matrix multiplication.]

In the above model A is called the coefficient matrix, X is called the variable matrix, and B is called the constant matrix.