Matrix Algebra II Video Lecture

Section 11.4

Course Learning Objectives:

1) **Solve systems of linear equations using technology.**
2) **Perform the algebra of matrices, find inverses of matrices, and solve matrix equations.**

Weekly Learning Objectives:

1) **Find the inverse of a matrix by hand and with a graphing utility.**
2) **Solve a system of linear equations using an inverse matrix.**

Matrix Algebra Part II

In a previous lecture we learned how to add, subtract, and multiply matrices if the dimensions were appropriate. In this section we learn how to divide matrices. The procedure in division of matrices is to multiply by the inverse of the matrix in the same way that we solve the equation $4x = 12$ by dividing both sides by 4 or equivalently multiplying both sides by $\frac{1}{4}$.

Before finding the inverse of a matrix we define the identity matrix, which plays the same role in matrix multiplication as 1 does for ordinary multiplication of numbers. Recall $1 \cdot a = a \cdot 1 = a$ for any number a.

The identity matrix I_n is the $n \times n$ matrix for which each main diagonal entry is a 1 and all other entries are 0.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ etc.}$$

Identity matrices behave like the number 1 in the sense that
$I_n \cdot$A $=$ A $\cdot I_n =$ A whenever the product is defined.

Check $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 4 & 1 \\ -1 & 0 & 2 & -3 \end{bmatrix} =$

If A and B are $n \times n$ matrices and if AB $=$ BA $= I_n$, then A & B are said to be inverses of each other. The concept of the inverse of a matrix is similar to that of the reciprocal of a number.

To find the inverse of an $n \times n$ matrix A:
(assuming the matrix has an inverse)
1) Form the matrix $[A|I_n]$
2) Transform the matrix $[A|I_n]$ into reduced row echelon form
3) The new matrix should look like $[I_n|A^{-1}]$ where $A^{-1}$is the inverse of A.

Example 1. Find the inverse of $\begin{bmatrix} 2 & -3 \\ 4 & -7 \end{bmatrix}$

Example 2. Find the inverse of $\begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}$ using your calculator.

Example 3. Find the inverse of $\begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}$

In the previous lecture we learned that a system of equations could be written as a matrix equation. For example the system

$$\begin{array}{ccccccc} x & - & 3y & + & 3z & = & 4 \\ 2x & + & y & - & 5z & = & -11 \\ x & - & 2y & + & z & = & 0 \end{array}$$

could be written $\begin{bmatrix} 1 & -3 & 3 \\ 2 & 1 & -5 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -11 \\ 0 \end{bmatrix}$. If we let

A $= \begin{bmatrix} 1 & -3 & 3 \\ 2 & 1 & -5 \\ 1 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, and B $= \begin{bmatrix} 4 \\ -11 \\ 0 \end{bmatrix}$ then the system

could be written as the matrix equation A$X =$ B. The matrix A is called the coefficient matrix. We can solve this system by multiplying each side of the equation by A^{-1}(providing the inverse exists).

$$
\begin{aligned}
\mathsf{A}X &= B \\
\mathsf{A}^{-1}(\mathsf{A}X) &= \mathsf{A}^{-1}\mathsf{B} && \text{Multiply on left by } \mathsf{A}^{-1} \\
(\mathsf{A}^{-1}\mathsf{A})X &= \mathsf{A}^{-1}\mathsf{B} && \text{Associative property} \\
I_3X &= \mathsf{A}^{-1}\mathsf{B} && \text{Property of inverses} \\
X &= \mathsf{A}^{-1}\mathsf{B} && \text{Property of Identity matrix}
\end{aligned}
$$

$\mathsf{A}^{-1} =$

$\mathsf{A}^{-1}\mathsf{B} =$

Therefore, the solution to the system is

Solve the system using an inverse matrix:

$$\begin{aligned} x + y \quad\quad &= 3 \\ -x + 3y + 4z &= -3 \\ 4y + 3z &= 2 \end{aligned}$$