

# Polynomials Video Lecture

## Section 4.1

### Course Learning Objectives:

- 1) Demonstrate an understanding of functional attributes such as domain, range, odd/even, increasing/decreasing, and symmetry. Determine these attributes for a function given its graph and/or its rule.
- 2) Graph polynomial functions and use such graphs to solve applied problems and to understand the significance of attributes of the graph to such applied problems.
- 3) Identify and articulate the significance of graphical components in a mathematical model/application.

### Weekly Learning Objectives:

- 1) Identify polynomial functions and their degree.
- 2) Identify the end behavior, symmetry, domain and range of a polynomial.
- 3) Graph polynomial functions using transformations.
- 4) Identify the real zeros of a polynomial function and their multiplicity.

# Polynomials

A polynomial function is a function constructed using the operations of addition, subtraction, and multiplication.

A polynomial function  $P$  of **degree  $n$**  is a function of the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

where  $a_n \neq 0$  and all exponents are positive integers.

The  $a_i$ 's are the coefficients,  $a_n$  is the leading coefficient, and  $a_0$  is the constant coefficient. Remember single termed polynomials are called monomials.

Almost all functions in mathematics and sciences can be evaluated by a polynomial. Most calculators use a polynomial approximation to calculate transcendental and other functions. For example, your calculator actually uses the polynomial below to calculate

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

Similarly,

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

These will be proven in Calculus.

Are there general characteristics that all polynomials have?

All polynomials are **smooth** and **continuous**. This means they have no breaks, holes, corners or cusps in their graphs. You can draw the entire graph without ever lifting your pencil.

On a calculator, let's look at the graphs of the following functions:

$$f(x) = x, \quad f(x) = x^2, \quad f(x) = x^3, \quad f(x) = x^4, \quad f(x) = x^5$$
$$f(x) = x^6, \quad f(x) = x^7, \quad f(x) = x^8$$

**Generalizations of the Power Function:**

$$f(x) = x^n$$

**Even Degree Polynomials:**

**End or Tail Behavior**

**Symmetry**

**Domain**

**Range**

**Odd Degree Polynomials:**

**End or Tail Behavior**

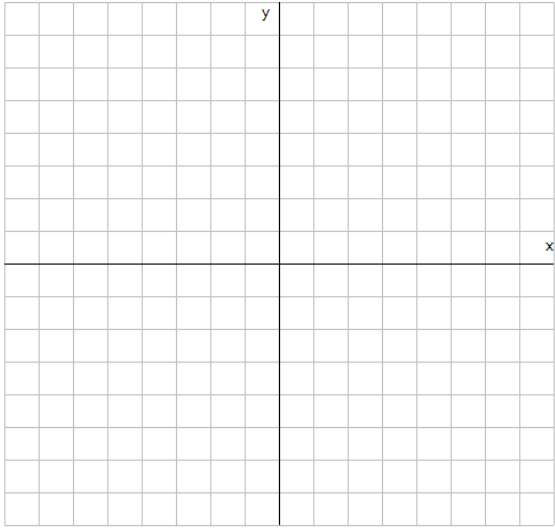
**Symmetry**

**Domain**

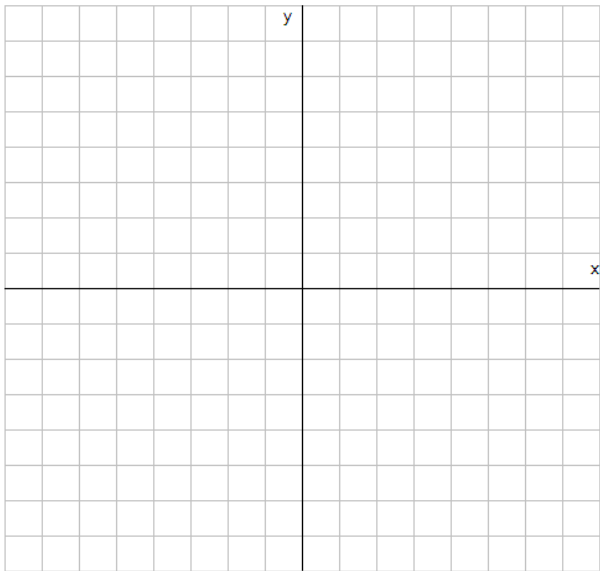
**Range**

Sketch the graph of the following:

$$f(x) = -(x-3)^5$$



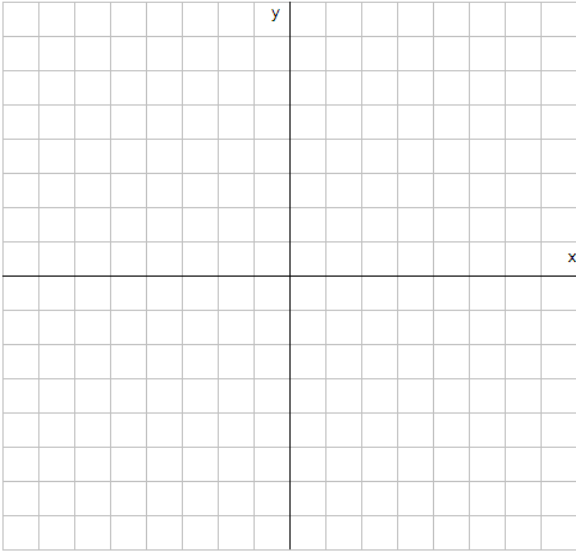
$$f(x) = -x^6 - 2$$



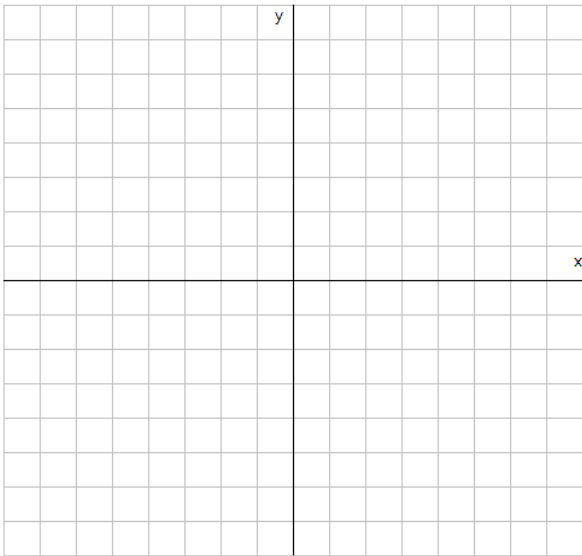
If  $P(x)$  is a polynomial and there exists a number  $c$  such that  $P(c) = 0$ , the  $c$  is a **zero** of  $P(x)$ . The zeros of the **solutions** or **roots** to the polynomial equation  $P(x) = 0$ . Also, if  $P(c) = 0$ , then  $x = c$  is an **x-intercept** of the graph of  $P(x)$ .

Find the zeros of the polynomials below and graph:

$$f(x) = x^2 - 3x - 4$$



$$g(x) = 3x^2 - x - 2$$



If  $P(x) = (x-3)^2(x+5)^3$ , we say that 3 is a zero of multiplicity 2 and -5 is a zero of multiplicity 3.

The following statements are all equivalent:

1.  $c$  is a zero of  $P(x)$
2.  $\{c\}$  is a root (solution) of the equation  $P(x) = 0$
3.  $x - c$  is a factor of  $P(x)$
4.  $(c, 0)$  is an intercept of the graph of  $y = P(x)$

Between any two zeros the graph of a polynomial will lie completely above or completely below the  $x$  - axis.

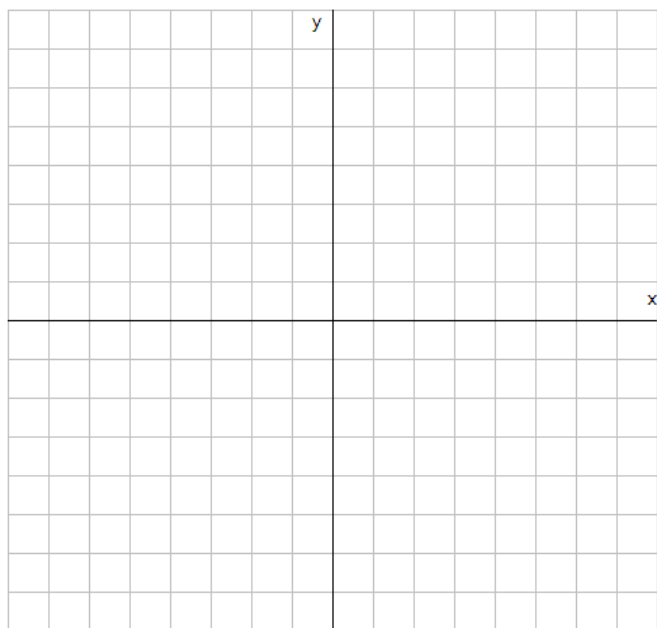
Characteristics of the graphs of generalized polynomials. i.e.

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

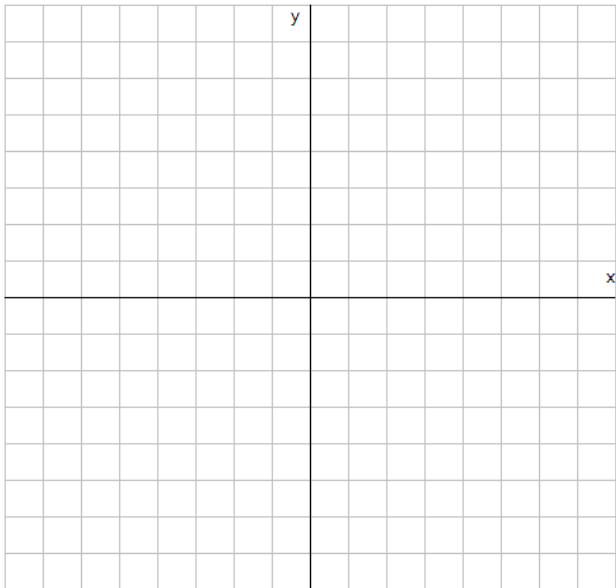
1. The graph is continuous, smooth, with no corners or cusps.
2. The graph will have at most  $n$   $x$  - intercepts.
3. The graph will have the same tail behavior as the tail behavior of the dominating term.
4. The graph will have at most  $n - 1$  relative extrema.

Sketch the graph of the following:

$$f(x) = (x-1)(x+3)(x)$$



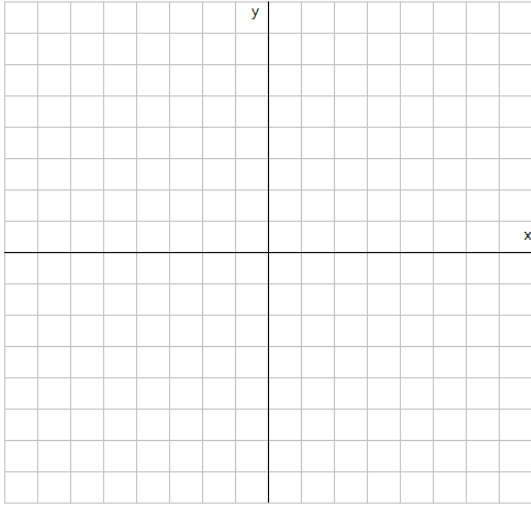
$$f(x) = (x+2)^2(x-1)$$



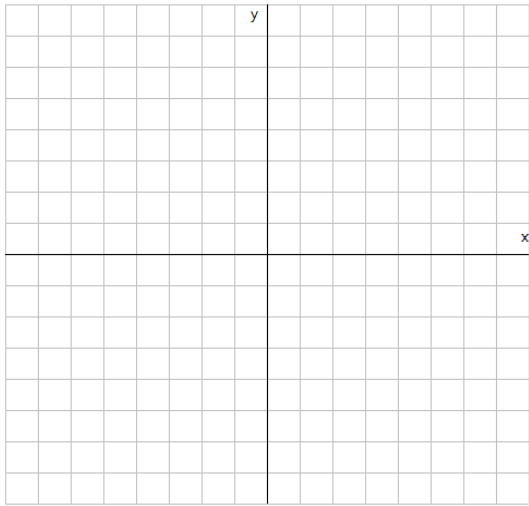
### Guidelines for graphing a polynomial function

1. Factor the polynomial to find all real zeros. The zeros are the intercepts of the graph. If the zero has even multiplicity, the graph will "bounce off" the x-axis and if the zero has odd multiplicity, then the graph will "pass through" the x-axis. Note: The function changes sign when the multiplicity is odd and does not change sign when the multiplicity is even.
2. Plot the x- and y-intercepts and note the multiplicity of the intercepts.
3. Determine the tail behavior.
4. Sketch a smooth curve that passes through the required points and has the appropriate end behavior.

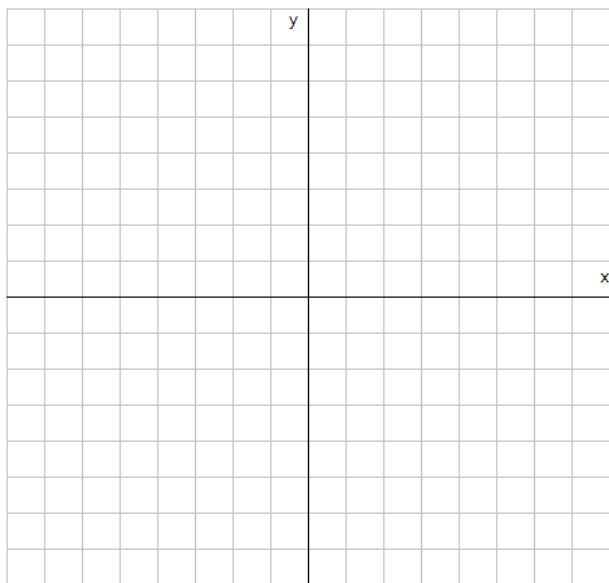
$$f(x) = (x+2)^3(x-1)^2$$



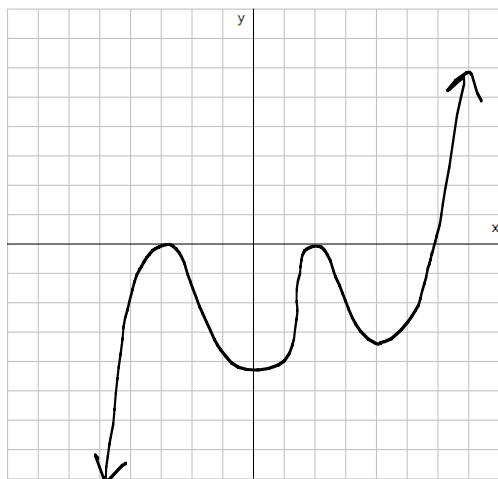
$$f(x) = (3x+2)^2(1-x)$$



$$f(x) = x(x-1)^2(4-3x)(x+4)^3$$



Could the following be the graph of a degree 3 polynomial?



Give a formula for a polynomial whose graph would have the following graph.

