

# Properties of Logarithms Video Lecture

## Section 5.5

### Course Learning Objectives:

**Demonstrate appropriate manipulation of exponential and logarithmic expressions. (Review from Math 940)**

### Weekly Learning Objectives:

- 1) Work with the properties of logarithms.**
- 2) Write a logarithmic expression as a single logarithm.**
- 3) Write a single logarithm as a sum or difference of logarithms.**
- 4) Use the change of base formula.**
- 5) Use a calculator to evaluate logarithms.**

## Properties of Logarithms

Properties of logarithms should parallel properties of exponents.

$$x^m \cdot x^n = \frac{x^m}{x^n} = (x^m)^n =$$

Properties of logarithms:

If  $a$  is a positive number, different from 1 and  $x$  &  $y$  are both greater than zero, and  $c$  is any real number, the following are true.

1.  $\log_a xy = \log_a x + \log_a y$
2.  $\log_a \frac{x}{y} = \log_a x - \log_a y$
3.  $\log_a (x^c) = c \log_a x$

Proof for 1 :

Examples: Write each of the following as a single logarithm. Assume that variables represent positive real numbers:

1.  $\log_2 9 + \log_2 3$

2.  $\log_6 (x^2 + 1) + \log_6 3 + \log_6 x$

3.  $\log_7 20 - \log_7 4$

4.  $\ln(a + b) + \ln(a - b) - 2 \ln c$

If  $\log_b 3 = .5$  and  $\log_b 5 = .7$ , find each of the following:

5.  $\log_b 25$

6.  $\log_b \frac{9}{5}$

7.  $\log_b \sqrt[3]{\frac{5}{27}}$

Write each expression as a sum or difference of logarithms:

8.  $\log_4 \frac{2}{9z}$

9.  $\log_b \sqrt{\frac{3}{y}}$

10.  $\log_b \frac{(x+5)^2}{x}$

11.  $\ln \frac{z\sqrt[4]{x}}{\sqrt[3]{y^2+6y+17}}$

Change of base formula:  $\log_b x = \frac{\log_a x}{\log_a b}$

How does  $\log_a b$  compare with  $\log_b a$ ?

Evaluate each of the following using your calculator:

**12.**  $\log 27$

**13.**  $\ln 142$

**14.**  $\log_6 40$

**15.**  $\log_8 100$

**16.** Find the exact value for  $(\log_2 5)(\log_5 7)$