

Properties of Trigonometric Functions Video Lecture

Section 6.3

Course Learning Objectives:

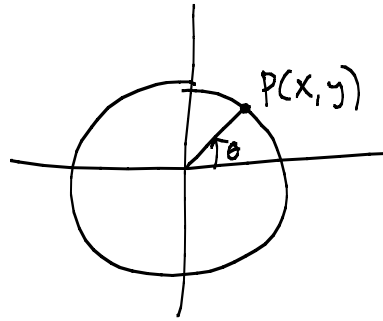
Demonstrate an understanding of trigonometric functions and their applications.

Weekly Learning Objectives:

- 1) Use a circle of radius r to evaluate the trigonometric functions.**
- 2) Determine the domain and range of the trigonometric functions.**
- 3) Determine the period of the trigonometric functions.**
- 4) Determine the signs of the trigonometric functions in a given quadrant.**
- 5) Find the values of the trigonometric functions using fundamental identities.**
- 6) Find the exact values of the trigonometric functions of an angle given one of the functions and the quadrant of the angle.**

Properties of Trigonometric Functions

Recall:



Let θ be any angle in standard position and let $P(x, y)$ be the terminal point on the unit circle that corresponds to θ .

$$\sin \theta = y \qquad \cos \theta = x \qquad \tan \theta = \frac{y}{x} \quad (x \neq 0)$$

$$\csc \theta = \frac{1}{y} \quad (y \neq 0) \qquad \sec \theta = \frac{1}{x} \quad (x \neq 0) \qquad \cot \theta = \frac{x}{y} \quad (y \neq 0)$$

If you are not on a unit circle, but rather θ is an angle in standard position on a circle of radius r where $x^2 + y^2 = r^2$ and $P = (x, y)$ is a terminal point on this circle, then

$$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r} \qquad \tan \theta = \frac{y}{x} \quad (x \neq 0)$$

$$\csc \theta = \frac{r}{y} \quad (y \neq 0) \qquad \sec \theta = \frac{r}{x} \quad (x \neq 0) \qquad \cot \theta = \frac{x}{y} \quad (y \neq 0)$$

For $\sin \theta$ and $\cos \theta$, θ can be any angle, so it follows that the domain of sine and cosine functions is the set of all real numbers.

Domain and Range of the trigonometric functions:

Function

$$f(\theta) = \sin \theta \quad \text{Domain:}$$

Range:

$$f(\theta) = \cos \theta \quad \text{Domain:}$$

Range:

$$f(\theta) = \tan \theta \quad \text{Domain:}$$

Range:

$$f(\theta) = \cot \theta \quad \text{Domain:}$$

Range:

$$f(\theta) = \sec \theta \quad \text{Domain:}$$

Range:

$$f(\theta) = \csc \theta \quad \text{Domain:}$$

Range:

The six trigonometric functions are all periodic:

Period of sine, cosine, secant and cosecant = 2π

Period of tangent and cotangent = π

Periodic Properties:

$$\sin(\theta + 2\pi) = \sin \theta \quad \cos(\theta + 2\pi) = \cos \theta$$

$$\tan(\theta + \pi) = \tan \theta \quad \cot(\theta + \pi) = \cot \theta$$

$$\sec(\theta + 2\pi) = \sec \theta \quad \csc(\theta + 2\pi) = \csc \theta$$

$x -$ $y +$	$x +$ $y +$
$x -$ $y -$	$x +$ $y -$

Signs of the trigonometric functions in quadrants:

Quadrant	Positive	Negative
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I

II

III

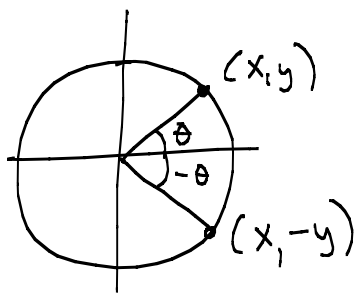
IV

To help you remember which trig functions are positive:

<u>Sine</u>	<u>All</u>
<u>Tangent</u>	<u>Cosine</u>

"All Students Take Calculus"

Trig functions - Are they even, odd, or neither?



$$\sin(-\theta) =$$

$$\cos(-\theta) =$$

$$\tan(-\theta) =$$

$$\csc(-\theta) =$$

$$\sec(-\theta) =$$

$$\cot(-\theta) =$$

Fundamental Identities - **MEMORIZE!!!!**

Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities - **MEMORIZE!!!!**

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$

Find the value of all of the trigonometric functions if you know that $\sin \theta = \frac{3}{4}$ and that θ is in quadrant II.

Solution 1: Using a unit circle

Solution 2: Using identities

Given that $\tan \theta = \frac{1}{2}$ and $\sin \theta < 0$, find the exact value of each of the remaining five trigonometric functions of θ .

Solution 1: Using a circle of radius r

Solution 2: Using identities

Express all trigonometric functions in terms of the cotangent function.