Right Triangle Trigonometry Video Lecture

Section 8.1

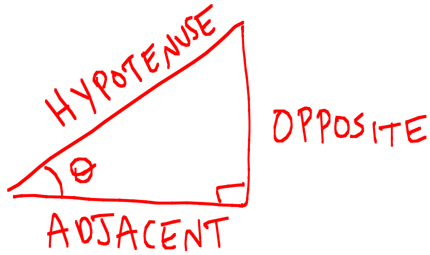
Course Learning Objectives:
Demonstrate an understanding of trigonometric functions and their applications.

Weekly Learning Objectives:

1) **Find the value of trigonometric functions of acute angles using right triangles.**
2) **Use the complementary angle theorem and find cofunction values.**
3) **Solve right triangles.**
4) **Solve applied problems involving angles of elevation and depression.**
5) **Find the exact values of the trigonometric functions of an angle given one of the functions and the quadrant of the angle.**
6) **Find the exact values of the trigonometric functions of an angle given the coordinates of the terminal point.**

Right Triangle Trigonometry

Consider a right triangle with θ as one of the acute angles.



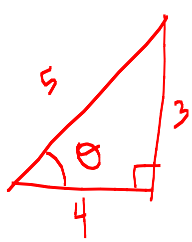
The trigonometric ratios are defined as follows **(SOHCAHTOA)**

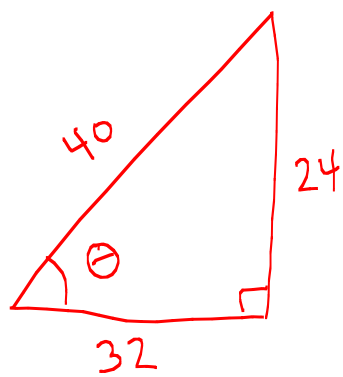
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$	$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$	$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

Note that the reciprocal relationships and the relationships between sin, cos, tan, and cot still hold as defined previously.

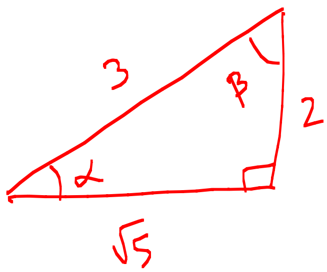
Since any two right triangles with angle θ are similar, the ratios are the same regardless of the size of the triangle.

Find the trig ratios for each of the triangles:





Compute the trigonometric ratios for angles α and β in the following triangle.



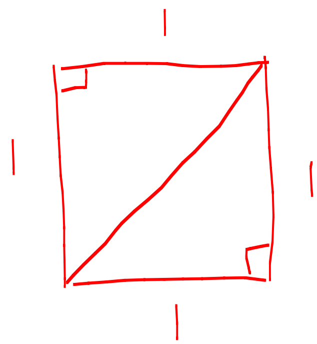
What is the relationship between α and β?

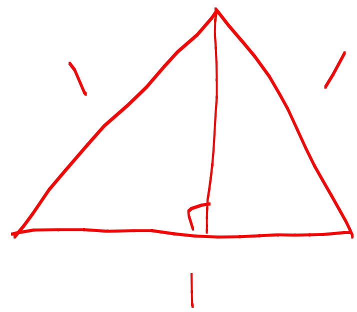
Complimentary Angle Theorem:

Express each of the following as a cofunction value:

sin $35°$ cot $\frac{\pi}{7}$ sec $25°$

Certain right triangles have ratios that can be calculated easily using the Pythagorean Theorem and these ratios lead to confirmation of some of the special values of the trig functions that we have memorized.





θ in degrees	θ in radians	**sin θ**	**cos** θ	**tan** θ	**csc** θ	**sec** θ	**cot** θ
$\mathbf{0^\circ}$	$\mathbf{0}$	0	1	0	—	1	—
$\mathbf{30^\circ}$	$\boldsymbol{\frac{\pi}{6}}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{2}$	$\sqrt{3}$
$\mathbf{45^\circ}$	$\boldsymbol{\frac{\pi}{4}}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\mathbf{60^\circ}$	$\boldsymbol{\frac{\pi}{3}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$\mathbf{90^\circ}$	$\boldsymbol{\frac{\pi}{2}}$	1	0	—	1	—	0

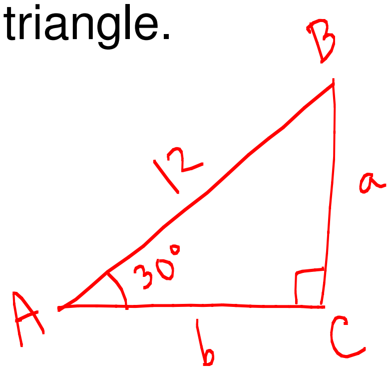
To find the values of the trigonometric ratios for other angles we use our calculators and the numerical methods that are programmed into them.

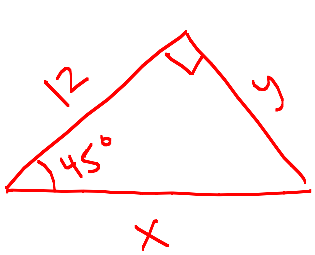
Note: We follow the convention that if there are no units on the measure of the angle, we assume that the measure is in radians. If the measure of the angle is in degrees, we will use the $°$ symbol. Therefore if we want to find the sin of 1 radian, we use the notation sin 1 and the value is $\approx .84147$. If we want to find the sin of 1 degree, we use the notation sin $1°$ and the value is $\approx .0175$.

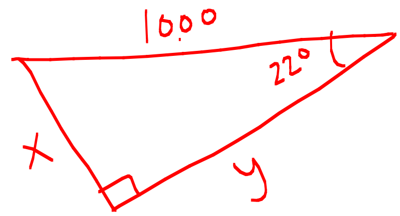
Find each of the following: cos $32°$ tan $.5$ sec $.1$

Many applied problems in navigation, surveying, astronomy, and measuring distances can be solved using right triangles and the trigonometric ratios.

To $solve$ a triangle means to find all 6 component parts of the triangle (3 sides & 3 angles) from the information given about the triangle.

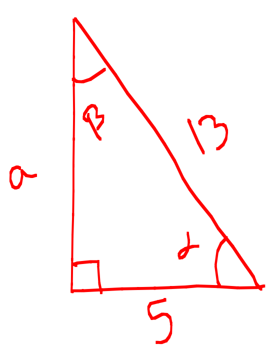




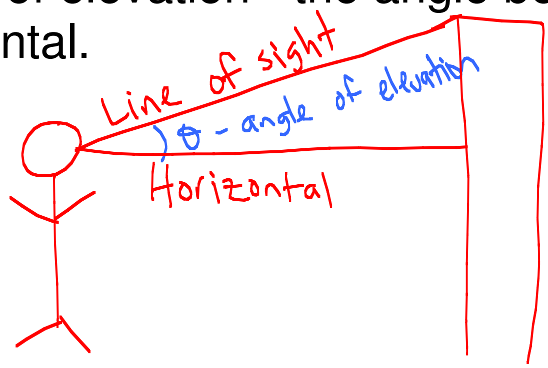


Sometimes we will need to find an angle of a triangle if the sides are known such as sin $\theta = \frac{1}{2}$. In this case clearly θ would need to be either $30°$ or $120°$ but since we have a right triangle, we know θ cannot be greater than $90°$. Therefore θ must be $30°$.

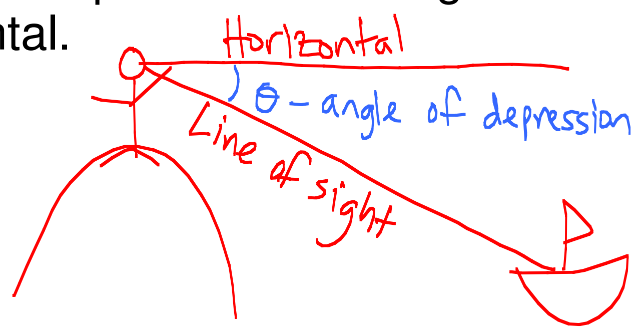
If we needed to find sin $\alpha = \frac{4}{5}$, this is a value that we have not memorized and we would need to use our calculator to undo the trigonometric function sin, i.e. we would need to find the inverse of the trig function sin. There is a key on the calculator indicated "invsin" or "sin^{-1}" that will undo the sin function. To find α using the calculator, we would enter sin$^{-1}(\frac{2}{3})$. If you want the value of θ in degrees, your calculator must be in degree mode. If you want the value of θ in radians, your calculator must be in radian mode.



Angle of elevation - the angle between the line of sight and the horizontal.



Angle of depression- the angle between the line of sight and the horizontal.



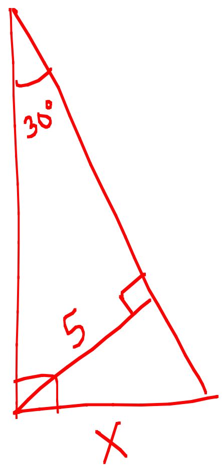
Examples:

A plane is flying at $35,000$ feet and estimates the angle of depression to a point on the ground at the base of a physical structure to be $22°$. What is the distance between the plane and the base of the physical structure?

A 600 foot guy wire is attached to the top of a communication tower. If the wire makes an angle of $65°$ with the ground, how tall is the tower?

A plane is flying at an elevation of 6000 feet, directly above a straight highway. Two motorists are driving cars on the highway on opposite sides of the plane, and the angle of depression to one car is 40° and to the other car is 65°. How far apart are the cars?

Find x in the following triangle.



Find the value of each of the trigonometric functions of α if α is in standard position and $(3, 7)$ is on the terminal side of angle α.

Find the value of each of the trigonometric functions of β if β is in standard position and $(-2, 5)$ is on the terminal side of angle β.

Find the value of the other trigonometric functions if $\sin\theta = -\frac{2}{3}$, $\theta \in QIII$.

Express tan θ in terms of sin θ, where θ is in quadrant II.