Solving Exponential and Logarithmic Functions Video Lecture

Section 5.6

Course Learning Objectives:
Solve certain types of exponential and logarithmic equations and inequalities. (Review from Math 940)

Weekly Learning Objectives:

1) Solve logarithmic equations.
2) Solve exponential equations.
3) Solve logarithmic and exponential equations using a graphing utility.
4) Solve logarithmic and exponential inequalities.

Solving Exponential and Logarithmic Equations

Solving exponential equations (Strategy)

1. If bases can be made the same, compare exponents and solve resulting equation.
2. If bases cannot be made the same, isolate variable term and take the logarithm of each side of equation.(Rewrite as a logarithm equation.)
3. Use properties of logarithms to rewrite exponential equation as a linear equation.
4. Solve accordingly.

Examples:

$3^x = 9^{5x-3}$

$\left(\frac{1}{8}\right)^{x^2} = 4^{\left(4x-\frac{3}{2}\right)}$

$2^{3x} = 34$

$10^{1-x} = 6^x$

$e^{3x^2-1} = 6$

$$\frac{8}{1 + e^{-x}} = 5$$

$3^{2x} + 3^x = 2$

$e^x - 12e^{-x} = 1$

Solving logarithmic equations (Strategy)

1. Isolate the logarithmic term on one side of the equation. You may have to combine logarithmic terms.
2. Rewrite equation in exponential form. (Take the exponential of each side i.e. Make each side of the equation the exponent on a common base)
3. Solve accordingly
4. Check solution to make sure it is in the domain of the original equation.

$\log_4(3x - 2) = 2$

$\log_5\left(x^2 + x + 4\right) = 2$

$\ln x = 8$

$\log_4 x + \log_4 (x - 3) = 1$

$2 \log x = \log 2 + \log(3x - 4)$

$\log_4(\log_2(x - 1)) = 0$

$$3^{\left(\frac{2}{\log_8 x}\right)} = 27$$

$$3x^2e^x + x^3e^x = 0$$

$$e^x = x^3 + 1$$

$$\ln x = x - 4$$

When solving a logarithmic or exponential inequality, take the logarithm or exponential of both sides (whichever is appropriate). If the function you are applying to both sides of the equation is increasing, the order of the inequality remains the same. If the function applied to both sides is decreasing, the order of the inequality reverses.

$3 \leq \log_{\frac{1}{2}} x < 5$

$e^{x^2} > e^{3x-2}$