Solving Trig Equations

Section 7.3

Course Learning Objectives:

1) Demonstrate an understanding of trigonometric functions and their applications.
2) Verify identities and solve trigonometric equations.

Weekly Learning Objectives:

1) Solve equations involving a single trigonometric function.
2) Solve trigonometric equations quadratic in form.
3) Solve trigonometric equations using identities.
4) Solve trigonometric equations linear in sine and cosine.
5) Solve trigonometric equations using a graphing utility.

Solving Trigonometric Equations

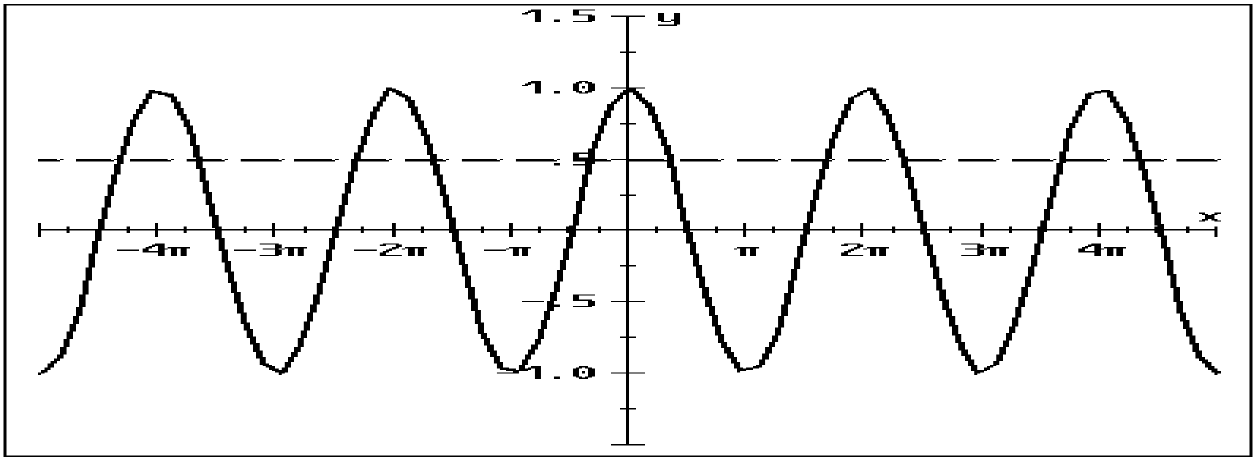
A trigonometric equation is an equation that contains trigonometric functions. $\sin 2x = 2\sin x \cos x$ and $2\sin x + 1 = 0$ are examples of trigonometric equations. The first is an and the second is a equation. The first is true for and the second is true for .

We have previously looked at ways of proving identities. We will now focus on finding solutions to conditional equations. Every trigonometric equation eventually reduces to an equation of the form $trig(x) = a$. For example : $\cos x = \frac{1}{2}$, $\tan x = -1$, $\csc x = 3$ are equations in this form.

To solve an equation that is in this form, we will either recall special values of the trigonometric functions that we have memorized or we will use the inverse function capabilities of our calculators.

Since each trigonometric function is periodic, there will be an infinite number of solutions to almost every trigonometric equation.

Consider the equation $\cos x = \frac{1}{2}$



What are the solutions in the interval $[0, 2\pi]$?

How could we represent <u>all</u> of the solutions?

For simplicity, we frequently restrict the intervals over which we report the solutions.

Examples:

Find all solutions in the interval $[0, 2\pi)$. Also find a representation for <u>all</u> of the solutions.

1. $\sqrt{2}\sin x - 1 = 0$

2. $\cot 2x - 1 = 0$

3. $\tan x = .3$

4. $\tan\left(x - \frac{\pi}{2}\right) = 1$

5. $\sec 5x = -2$

6. $\sin x = -.8$

Most trigonometric equations are more complicated than the form $trig(x) = a$, but, through the use of trigonometric identities and algebraic manipulations and equation solving properties, they can be converted into an equation or equations of the form $trig(x) = a$.

The given equation may be quadratic, radical, or fractional in nature in which case we would solve accordingly. Recall the situations in which you might introduce extraneous solutions.

The equation may also involve multiple trigonometric functions and/or different arguments. A subgoal in solving more complicated trigonometric equations is to get an equation in a form involving a single trigonometric function of a single number.

Additional examples:

Find all solutions in the interval $[0, 2\pi)$.

7. $\csc^2 x - 4 = 0$

8. $2\cos^2 x + \cos x = 1$

9. $\sin 2x \sin x = \cos x$

10. $\csc^2 x = \cot x + 1$

11. $\cos 2\theta + 5\cos\theta + 3 = 0$

12. $5\tan^3 x - 5\tan^2 x - \tan x + 1 = 0$

13. $\tan\dfrac{x}{2} - \sin x = 0$

14. $\cos 5x - \cos 7x = 0$ Hint: Factor using a sum to product identity.

15. $\cos x \cos 2x + \sin x \sin 2x = -\dfrac{1}{5}$

16. $\sqrt{3}\sin\theta + \cos\theta = 1$

17. $\sin x = \ln x$