The Zeros of a Polynomial Video Lecture

Sections 4.2 and 4.3

Course Learning Objectives:

1) Graph polynomial functions and use such graphs to solve applied problems and to understand the significance of attributes of the graph to such applied problems.
2) Solve appropriate applications of determining zeros of polynomials.

Weekly Learning Objectives:

1) Use the rational zeros theorem to list the potential real zeros of a polynomial function.
2) Find the exact value of all zeros of a polynomial function.
3) Factor polynomials completely.
4) Use the Boundedness Theorem.

The Zeros of a Polynomial

Theorem: A polynomial function of degree n ($n \geq 1$, has at most n real zeros.
(We count each zero as many times as its multiplicity.)

$$P(x) = x^3 - x^2 - 14x + 24 = (x-3)(x-2)(x+4)$$

The following theorem gives us a list of possible rational zeros for a polynomial function if the coefficients are integers.

Rational Zeros Theorem: If the polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

has integer coefficients, then every rational zero of P(x) is of the form $\frac{p}{q}$
where p is a factor of a_0 **and q is a factor of** a_n.

List the possible rational zeros for f(x) = $2x^3 + x^2 - 13x + 6$
Find all the real zeros.

List the possible rational zeros for f(x) = $x^4 - 5x^3 - 5x^2 + 23x + 10$
Find the real zeros.

Boundedness Theorem: Let P(x) be a polynomial with real coefficients.

1. **If we divide P(x) by x - b using synthetic division, and if the row that contains the quotients and the remainder has no negative entry, then b is an upper bound for the real roots of P(x) = 0.**

2. **If we divide P(x) by x - b using synthetic division, and if the row that contains the quotients and the remainder has entries that are alternately nonpositive and nonnegative then b is a lower bound for the real roots of P(x) = 0.**

Show that -3 and 5 are the lower and upper bounds for the real roots of

$$x^4 - 2x^3 - 9x^2 + 2x + 8 = 0$$

Find the exact value of all zeros of g(x) = $7x^3-32x^2+44x-16$

Find the exact value of all zeros of f(x) = $x^4-6x^3+4x^2+15x+4$

Find the exact value of all zeros of g(x) = $2x^6-3x^5-13x^4+29x^3-27x^2+32x-12$

We know that nth degree polynomial can have at most n real zeros. In the complex number system an nth degree polynomial has exactly n zeros (counting multiplicity). This is called the **Fundamental Theorem of Algebra**.

Find all zeros of P(x) = $x^3 - 8$ and factor it completely.

Find all zeros of P(x) = $x^4 + 10x^2 + 25$ and factor it completely.

Find all zeros of P(x) = $x^3 - x^2 + x$ **and factor it completely.**

Find all zeros of P(x) = $3x^5 + 24x^3 + 48x$ **and factor it completely.**

Find all zeros of P(x) = $x^5 - x^4 - 4x^3 + 8x^2 - 32x + 48$ and factor it completely.

Find all zeros of P(x) = $3x^4 + 5x^3 + 25x^2 + 45x - 18$ and factor it completely.